restrictions of wellfounded relations

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```
In[1]:= SetDirectory["l:"]; << goedel74.11b; << tools.m

        :Package Title: goedel74.11b            2005 October 11 at 5:00 p.m.

        It is now:  2005 Oct 12 at 21:24

        Loading Simplification Rules

        TOOLS.M                       Revised 2005 October 11

        weightlimit = 40
```

summary

Any subset of a well-founded relation is well-founded. In the opposite direction, a relation is well-founded if all its restrictions to sets are well-founded. The same goes for well-orderings: a relation is well-ordered if all its restrictions to sets are well-ordered.

derivation

The derivation of the result for well-founded relations can be done in a single step as follows:

```
In[2]:= SubstTest[subclass, image[inverse[S], u], image[inverse[S], v],
          {u -> image[IMAGE[id[x]], image[CART, Id]], v -> WF}] // Reverse

Out[2]= subclass[image[IMAGE[id[x]], image[CART, Id]], WF] == WELLFOUNDED[composite[Id, x]]

In[3]:= subclass[image[IMAGE[id[x_]], image[CART, Id]], WF] := WELLFOUNDED[composite[Id, x]]
```

A lemma is needed for the case of well-orderings.

```
In[4]:= SubstTest[and, TOTALORDER[z], WELLFOUNDED[intersection[Di, z]], z → composite[Id, x]]

Out[4]= and[TOTALORDER[composite[Id, x]], WELLFOUNDED[intersection[Di, x]]] ==
         WELLORDER[composite[Id, x]]

In[5]:= and[TOTALORDER[composite[Id, x_]], WELLFOUNDED[intersection[Di, x_]]] :=
         WELLORDER[composite[Id, x]]
```

Theorem.

```
In[6]:= SubstTest[subclass, u, intersection[v, w], {u → image[IMAGE[id[x]], image[CART, Id]],
          v → TO, w → image[inverse[IMAGE[id[Di]]], WF]}]

Out[6]= subclass[image[IMAGE[id[x]], image[CART, Id]], WO] == WELLORDER[composite[Id, x]]

In[7]:= subclass[image[IMAGE[id[x_]], image[CART, Id]], WO] := WELLORDER[composite[Id, x]]
```