zero products

Johan G. F. Belinfante
2002 October 4

```
<< goedel52.p72; << tools.m

:Package Title: goedel52.p72          2002 October 3 at 11:25 p.m.

It is now:  2002 Oct 4 at 15:5

Loading Simplification Rules

TOOLS.M                   Revised 2002 August 30

weightlimit = 40
```

■ summary

A rewrite rule is derived for a product of natural numbers to be zero. The technique used is to look at horizontal and vertical sections of the natural multiplication function **NATMUL**. Both of these come out in terms of iterated iterations. By substituting the one formula into the other, the iterated iteration expression cancels out, leaving us with the expected result, but without any real clue about how the proof goes. The **GOEDEL** program does everything behind the scenes: it is not a program for finding the proofs of theorems, but only a tool for discovering facts. Those facts will still need to be proved using **Otter**, or some other theorem prover.

■ derivation

Lemma: (horizontal sections)

```
SubstTest[member, pair[x, y], image[inverse[w], singleton[0]], w → NATMUL] // Reverse

and[member[x, omega],
  member[pair[y, 0], iterate[iterate[SUCC, singleton[x]], singleton[0]]]] ==
 or[and[equal[0, x], member[y, omega]], and[equal[0, y], member[x, omega]]]
```

This provides a formula for eliminating the **iterate[iterate[SUCC,singleton[x]]]** expression:

```
and[member[x_, omega],
  member[pair[y_, 0], iterate[iterate[SUCC, singleton[x_]], singleton[0]]]] :=
 or[and[equal[0, x], member[y, omega]], and[equal[0, y], member[x, omega]]]
```

Now when the same expression appears in the formula for a vertical section, it is automatically eliminated, and we are left with a simple formula

```
SubstTest[member, 0, image[w, cart[singleton[x], singleton[y]]], w → NATMUL]

equal[0, natmul[x, y]] ==
 or[and[equal[0, x], member[y, omega]], and[equal[0, y], member[x, omega]]]
```

This equation will be added as a new permanent rewrite rule:

```
equal[0, natmul[x_, y_]] :=
 or[and[equal[0, x], member[y, omega]], and[equal[0, y], member[x, omega]]]
```

The new rule states a necessary and sufficient condition for **natmul[x,y]** to be zero, replacing a weaker existing rule.