# Money, Inflation, and Trading Behavior: Theory and Laboratory Experiments * 

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#### Abstract

In a Kiyotaki-Wright economy, we generated equilibria with varying levels of productivity and welfare by adjusting the quantity of fiat money and inflation. Agents coordinate on a speculative equilibrium with elevated productivity when inflation is low and the quantity of money is limited. Inflation can drive the economy towards a fundamental equilibrium, by reducing people's trust in fiat money. For instance, when the inflation rate goes from 0 to 8 percent, aggregate welfare declines about one-third. The liquidity of traded commodities plays a crucial role in determining the equilibrium selection. We adapted the model to a laboratory-like setting with a small number of agents and found similar relationships. To test the theoretical predictions, we conducted laboratory experiments with real people. Our experimental data show that inflation reduces trust in fiat money and that an increase in the quantity of money may decrease speculative behavior. However, we did not observe any significant influence of inflation on trading strategies, a result that suggests inflation has minimal welfare effects.


Keywords: Inflation, Speculative Equilibrium, Acceptability of Money, Laboratory Experiments

JEL Codes: C63, C91, D83, E41

[^0]
## 1 Introduction

The recent monetary expansion in many advanced countries has reignited interest among scholars and policymakers in understanding the effect of money creation and inflation on individuals' purchasing and trading behaviors and how these responses are transmitted to the macroeconomy. Policymakers in the USA and Europe are concerned when the inflation rate goes above an ideal target of $2 \%$. A common concern is that inflation can lead to socially wasteful activities as individuals attempt to shift the inflation tax onto others (see, among others, Lucas, 2000; Lagos and Rochetau, 2005; and Cooley and Hansen, 1989). Another concern is its redistributive effects, as inflation disproportionately affects individuals with a larger share of their wealth in cash and those employed in cash-intensive sectors. However, once the economy has reached an equilibrium with a higher-than-ideal rate of inflation, attempts to reduce it through monetary policies can have the unintended consequence of slowing down economic growth.

In this paper, we explore the trading reaction of people to money and inflation and the resulting aggregate productivity effects in an extension of Kiyotaki and Wright (1989, henceforth KW). In this set-up, producers and traders can coordinate on a speculative equilibrium, characterized by elevated aggregate productivity, or on a fundamental equilibrium that has a lower aggregate productivity. We want to comprehend the effect of money quantity and inflation rate on the formation of a fundamental or a speculative equilibrium. The emergence of a speculative equilibrium is facilitated by the trust people have in fiat money. In such equilibrium, liquidity considerations lead some individuals to accept commodities in trade that are relatively costly to hold. An advantage of the KW framework is that the microeconomic environment is intuitive when implemented in a laboratory settings. Yet, the model gives rise to different equilibria, each with a different level of production. One may then ask what is the role of the quantity of money and inflation in determining the emergence of a particular equilibrium and whether the observed trading strategies align with the equilibrium predictions.

We proceed in four steps. First, we expand the KW model to incorporate an inflation tax and examine the influence of the amount of fiat money and inflation on the formation of various equilibria. We also assess the welfare implications of these equilibria. Second, we adapt the original model, which assumes a continuum of agents, to an environment with a limited number of individuals, reflecting the types of interactions observed in laboratory experiments. Third, we report how individuals responded to changes in the supply of money and inflation in our laboratory experiments, focusing on their acceptance of money and speculative trading strategies. Finally, we use statistical analysis to analyze and interpret the results of the experiments.

We introduce inflation as a seigniorage tax, which involves the government randomly confiscating fiat currency from the general population. This results in money becoming a "hot potato" in circulation. Because individuals are specialized in production and consumption, inflation affects not only those who hold cash, but can also alter the frequency of trade in commodities and the flow of aggregate production. Earlier works by Duffy and Ochs (1999, 2002) and Duffy (2001) used a similar KW framework to explore in laboratory experiments the acceptance of money and the emergence of speculative equi-
libria. Our analysis differs from these works in two important ways. First, we include inflation. Second, we center the analysis on how individuals trading strategies respond to variations of aggregate variables, such as the liquidity of fiat money, the liquidity of the cheap-to-carry commodity, or inflation, rather than changes in their payoffs. Indeed, we keep consumption utility, production cost, and storage costs constant throughout the experiments. The environment consists of three reproducible and durable goods, and a non-reproducible object that serves as fiat money. Trade takes place in a decentralized setting through random, anonymous pairwise matching. The lack of record keeping and monitoring technology makes money "essential". We parameterize the model, in terms of consumption utility, production cost, and inventory costs, in a way that speculative and fundamental equilibria with complete or partial acceptance of fiat money emerge by varying the quantity of money and the rate of inflation. For instance, under no inflation, when the stock of money is low, the model predicts that individuals coordinate on a speculative equilibrium with a complete acceptance of money. When the stock of money is large, however, it is the a fundamental equilibrium that emerges. Although some microeconomic mechanisms that link inflation, quantity of money and aggregate productivity are specific to the KW model, the insights echo the popular view in monetary economics that an excess of money supply and inflation slow down economic activities. ${ }^{1}$

An immediate comparison of laboratory experiments with the baseline model's predictions can be misleading. The strategic interaction among an infinite number of agents, can lead to a different outcome than that among a limited number of individuals. Additionally, the baseline model assumes that agents are rational and forward-looking. These traits may not characterize accurately those of people in laboratory experiments. To address these concerns, we adapted the baseline model to an economy with a finite number of agents and then used it to simulate exchanges among computer-agents with the same number of participants and payoffs as in the laboratory experiments. We found that the distribution of goods and money in the computer-simulated experiments is similar to the corresponding Nash equilibria with a continuum of agents. This outcome facilitates our comparison between model's predictions and results from human subject experiments.

Another challenge in our study was the discrepancy between the model's assumption of an infinite time horizon and the short-time horizon of our experiments. Because fiat money, which has no consumption value, becomes worthless at the end of the game, individuals are less likely to accept it if the time horizion is short. To deal with this discrepancy, we followed the experimental literature on infinitely repeated games and used a probabilistic continuation rule. This transforms an infinitely repeated game into one played within a finite time frame, as described in studies such as Duffy and Ochs (1999) and Rustichini and Villamil (2000). The continuation rule is based on the premise that a constant continuation probability is equivalent to a constant time-discount rate

[^1]in an infinite horizon, assuming individuals are risk-neutral. Jiang, Puzzello, and Zhang (2023) present an alternative approach to the same issue by modifying the theoretical environment to sustain fiat money in equilibrium even with a finite horizon.

The results of our experiment indicate that inflation negatively affects people's trust in fiat money and that an increase in the money supply in some circumstances decreases speculative behavior. Nevertheless, we did not find any significant effect of inflation on trading strategies, which suggests that its welfare effects are modest.

There has been a growing body of literature that examines monetary issues in experimental settings. Earlier studies on inflation adapted overlapping generation models for laboratory use (Marimon and Sunder 1993, Lim et al. 1994, Bernasconi and Kirchkamp 2000). A more recent literature has looked into how key equilibrium predictions of search models are affected by monetary policy (Lagos and Wright 2005, Molico 2006, Aruoba et al. 2007, Craig and Rocheteau 2008, Duffy and Puzzello 2022, and Jiang, Puzzello and Zhang 2023). Our paper is part of a larger experimental literature that analyzes the role of money as a medium of exchange (Brown 1996, Duffy and Ochs 1999, 2002, Camera et al. 2003, Camera and Casari 2014, Duffy and Puzzello 2014, Jiang and Zhang 2018, Rietz 2019, Ding and Puzzello 2020).

## 2 The Model

The model economy is similar to that of KW, with the addition of an inflation tax. The time is divided into discrete periods. The economy is inhabited by infinitely-lived agents of mass one. There are three types of individuals and three types of goods, both denoted by $i=1,2,3$. The population is equally divided among the three types. A type $i$ individual consumes only goods of type $i$, and is specialized in producing goods of type $i+1$ (modulo 3). Production takes place immediately after consumption. Each unit of consumption produces a utility of $U$, while the cost of producing each unit is $D$. The sequence of consumption and production then yields a net utility of $u=U-D$. Goods are indivisible and durable. A unit of type $i$ good can be stored at a cost of $c_{i}$ per period. In addition to commodities, agents can also hold fiat money, $m$, at no cost. Money serves as a means of transaction but does not bring any utility in itself. An individual can hold either one unit of a type $i$ good or one unit of money, but not both. This simplifies the analysis and makes the decision to accept money more transparent. The fraction of the population holding money is equal to the overall stock of money, $Q$. There is a significant body of work dealing with asset-holding restrictions, such as those described here (e.g., Cavalcanti and Wallace, 1999; Duffie et al., 2005). A common discount factor of $0<1-\rho<1$ is applied between each period, but not within a period. At the start of each period, the government collects a tax from money holders. With a probability of $\delta_{m}$, a money holder pays a tax of one unit and then immediately produces one unit of commodity at a cost $D .{ }^{2}$ The government uses the collected tax revenue to purchase
${ }^{2}$ See Li (1994, 1995) and, more recently, Deviatov and Wallace (2014) and Bonetto and Iacopetta (2019) for a similar modeling of inflation in a KW environment, and Duffy and Puzzello (2022) and Jiang et al. (2023) for the treatment of inflation in a Lagos and Wright (2005) framework.
goods, on a one-to-one basis, from agents selected at random. This results in a quantity of money, $Q$, that remains constant over time. Afterwards, agents pay storage costs if they are holding commodities. Then, agents are randomly and uniformly paired for bilateral trade. Two parties engage in trade if and only if they both agree to the exchange. A type $i$ always accepts good $i$ and consumes it immediately. Therefore, a type $i$ trader arrives at the trade meeting with either with good $i+1$, or good $i+2$, or $m$. The choice of type $i$ individuals to trade good $j$ for good $k$ is denoted with $s_{j, k}^{i}=1$; otherwise $s_{j, k}^{i}=0$.

### 2.1 Commodities, Fiat Money, and Trading Strategies

This section briefly describes the evolution of the stock of commodities and money and the optimizing trading strategies.
Distribution of Commodities and Fiat Money. We denote with $p_{i, j}(t)$ the proportion of type $i$ agents that hold good $j$ at time $t$. Since $p_{i, i}(t)=0$, we have that

$$
\begin{equation*}
p_{i, i+1}(t)+p_{i, i+2}(t)+p_{i, m}(t)=\frac{1}{3} \tag{1}
\end{equation*}
$$

The following equation accounts for the overall holding of fiat money:

$$
\begin{equation*}
p_{1, m}(t)+p_{2, m}(t)+p_{3, m}(t)=Q \tag{2}
\end{equation*}
$$

The state of the economy at time $t$ can then be represented by the five-dimensional vector $\mathbf{p}(t)=\left(p_{1,2}(t), p_{2,3}(t), p_{3,1}(t), p_{1, m}(t), p_{2, m}(t)\right)$. The Online Appendix A. 1 details the evolution of $\mathbf{p}$, for a given set of strategies $s_{j, k}^{i}$ and an initial state $\mathbf{p}(0)$. The property of the dynamics of a similar environment are also studied in Iacopetta (2019) and Bonetto and Iacopetta (2019). We say that $\hat{\mathbf{p}}$ is a steady state for the strategies $s_{j, k}^{i}$ if $\mathbf{p}(t)=\hat{\mathbf{p}}$ for every $t$.
Strategies. Individuals are aware of the state of the economy, denoted by $\mathbf{p}$, and consider the strategies of all other individuals, including those of their own type, as fixed. The expected discounted utility of a type $i$ agent a time $t$, playing strategy $\sigma_{j, k}^{i}$ is:

$$
\begin{equation*}
V_{i, j}(t)=\sum_{t}^{\infty}(1-\rho)^{(\tau-t)} \sum_{l} \pi_{l, j}^{i}(\tau, t) v_{i, l}(\mathbf{p}(\tau)) d \tau \tag{3}
\end{equation*}
$$

where $\pi_{l, j}^{i}(\tau, t)$ is the probability that the individual will hold good $l$ at time $\tau \geq t$, given that he carries good $j$ at time $t$ and plays strategy $\sigma_{j, k}^{i}$. The term $v_{i, l}(\mathbf{p})$ is the flow of utility, net of storage costs, associated to the distribution of holdings $\mathbf{p}$. Note that both $\pi_{l, j}^{i}(\tau, t)$ and $v_{i, l}(\mathbf{p})$ are dependent on the strategies of all other individuals. In a steady state, the value functions, $V_{i, j}$, are linked to the inventory distribution, $\mathbf{p}$, through a system of linear equations. Given the strategies of the rest of the population, $\sigma_{j, k}^{i}$ maximizes the expected flow of utility of a type $i$ if and only if

$$
\sigma_{j, k}^{i}= \begin{cases}1 & \text { if } \Delta_{j, k}^{i}<0  \tag{4}\\ 0 & \text { if } \Delta_{j, k}^{i}>0 \\ 0.5 & \text { if } \Delta_{j, k}^{i}=0\end{cases}
$$

where $\Delta_{j, k}^{i} \equiv V_{i, j}-V_{i, k}$. Eq. (4) implies that $\sigma_{j, k}^{i}=1-\sigma_{k, j}^{i}$ and $\sigma_{j, j}^{i}=0$. Thus, that the full set of strategies for a type $i$ agent simplifies to $\boldsymbol{\sigma}^{i}=\left(\sigma_{i+1, m}^{i}, \sigma_{i+2, m}^{i}, \sigma_{i+1, i+2}^{i}\right)$, where $\boldsymbol{\sigma}^{i} \in \Sigma=\{(1,1,1),(1,0,1),(1,1,0),(0,1,0),(0,0,1),(0,0,0)\} .{ }^{3}$ The $3 \times 3$ matrix $\boldsymbol{\sigma}=\left(\boldsymbol{\sigma}^{1}, \boldsymbol{\sigma}^{2}, \boldsymbol{\sigma}^{3}\right) \in \Sigma^{3}$ summarizes all the strategies of the three types of agents.

### 2.2 Nash Equilibrium

A steady state Nash equilibrium is a time-invariant set of strategies that maximizes individuals' payoffs. Denote the set of strategies for the population with $\mathbf{s}=\left(\mathbf{s}^{1}, \mathbf{s}^{2}, \mathbf{s}^{3}\right) \in \Sigma^{3}$, where $\mathbf{s}^{i}=\left(s_{i+1, m}^{i}, s_{i+2, m}^{i}, s_{i+1, i+2}^{i}\right) \in \Sigma$, and the best responses of the three types of individuals with $\boldsymbol{\sigma}=\left(\boldsymbol{\sigma}^{1}, \boldsymbol{\sigma}^{2}, \boldsymbol{\sigma}^{3}\right) \in \Sigma^{3}$ where $\boldsymbol{\sigma}=\boldsymbol{\mathcal { B }}(\mathbf{s})$. The set of strategies $\mathbf{s}^{*}$ is a Nash equilibrium if $\boldsymbol{\sigma}=\mathbf{s}^{*}=\boldsymbol{\mathcal { B }}\left(\mathrm{s}^{*}\right)$.

Familiar properties of KW Nash equilibria are easy to verify when there is no fiat money in the economy, that is when $Q=0$. For instance, if $c_{1}<c_{2}<c_{3}$, two equilibria exist: a fundamental equilibrium characterized by the strategy triplet $\left(s_{2,3}^{1}, s_{1,3}^{2}, s_{1,2}^{3}\right)=$ $(0,1,0)$, and a speculative equilibrium, where the triplet is $(1,1,0)$. Only the strategies of type 1 individuals differ in the two equilibria. The fundamental equilibrium occurs when $\left(c_{3}-c_{2}\right) / u>1 / 6$; otherwise, the speculative equilibrium arises. In the latter, type 1 agents trade good 2 (with low storage cost) for good 3 (with high storage cost) due to liquidity considerations; good 3 is more likely than good 2 to be accepted in future trades for good 1.

When the supply of money is positive, $Q>0$, these two equilibria can be associated with full or partial acceptance of money. The matrix

$$
\mathbf{s}=\left(\begin{array}{ccc}
s_{2, m}^{1} & s_{3, m}^{2} & s_{1, m}^{3} \\
s_{3, m}^{1} & s_{1, m}^{2} & s_{2, m}^{3} \\
s_{2,3}^{1} & s_{1,3}^{2} & s_{1,2}^{3}
\end{array}\right)=\left(\begin{array}{ccc}
1 & 1 & 1 \\
1 & 0 & 1 \\
1 & 1 & 0
\end{array}\right)
$$

depicts a speculative equilibrium (see the last row of the matrix) where fiat money is generally accepted, except for type 2 individuals who possess good 1 (indicated by the 0 in the middle row). This scenario may arise when inflation is high. In this case, type 2 individuals prefer to pay the storage costs of holding good 1 rather than holding onto fiat money, so as to avoid paying the inflation tax.

[^2]
### 2.3 Implications and Hypotheses

In anticipation of the experimental tests, here we discuss further the microeconomic mechanisms that lead the economy to move from one equilibrium to another when the quantity of money or the inflation rate is varied. We characterize the changing conditions in the economy through the liquidity of money and of the three commodities. We measure liquidity through an index of acceptability, which refers to the proportion of times an object is accepted in trade compared to the number of times it is offered. Specifically, let $o_{i}(s)$ be the probability that good $i$ is offered (but not necessarily traded) on the market at time $s$, then $o_{i}(s)=\sum_{k, k^{\prime}} \sum_{j} p_{k, i} p_{k^{\prime}, j} s_{i, j}^{k}$. Let $t_{i}(s)$ be the probability that good $i$ is traded on the market at time $s$, then $t_{i}(s)=\sum_{k, k^{\prime}} \sum_{j} p_{k, i} p_{k^{\prime}, j} s_{i, j}^{k} s_{j, i}^{k^{\prime}}$. Hence, the acceptability $a_{i}$ of good $i$ is defined as $a_{i}(v)=t_{i}(v) / o_{i}(v)$, for $i=1,2,3, m$. To understand how liquidity interacts with the individual's payoff to determine the optimal strategy, we also follow the value function differentials of two relevant assets. For instance, if we are interested in the switch between the speculative and the fundamental equilibrium, we try to understand how money or inflation can turn $\Delta_{2,3}^{1}=V_{1,2}-V_{1,3}$ from negative into positive. Table 1 reports the parameters used throughout the paper. We set a relatively high production cost, $D$, to ensure that agents' decisions to hold money are sensitive to inflation - recall that production occurs immediately after paying the inflation tax. Note that inflation is costly for an individual not only because of the production cost, $D$, but also because the value of holding money is typically larger than that of holding a commodity.

Fig. 1a gives an overview of the steady state equilibria that occur over a range of fiat money $Q \in[0,0.9]$ and an inflation tax rate $\delta_{m} \in[0,0.09]$. The figure focuses on type 1's fundamental and speculative behavior and on type 2's acceptability of money. It shows that in the region with low or no inflation, all trader types value fiat money more than any commodity, regardless of the quantity of money in circulation. In this low-inflation region, a speculative equilibrium emerges at low levels of money, and a fundamental one at high levels of money. A similar pattern of speculative and fundamental equilibria exists for a moderate inflation rate, but type 2 agents no longer accept fiat money against good 1. Additionally, it is possible for a full-money-acceptance speculative equilibrium to coexist with a fundamental equilibrium where type 2 agents do not accept fiat money in an exchange with good $1 .{ }^{4}$ Fig. 3a shows that the liquidity of the three commodities at zero inflation border of fig. 1a declines with $Q$. Observe that the liquidity of good 3 drops more significantly than that of good 2 . One important implication of this development is shown in fig. 3b: As the quantity of money increases type 1's evaluation of good 2 relative to good 3 increases. We will then use the laboratory data to test the following statement:
Hypothesis 1 (H1): The incentives for type 1 agents to play speculative strategies weakens with an increase in the quantity of fiat money.

Fig. 2b also demonstrates that, along the zero-inflation border of fig. 1a, the evaluation of fiat money relative to that of other commodities drops, although mildly, as the quantity of fiat money increases. Hence we test that:

[^3]Hypothesis 2 (H2): The evaluation of money decreases with the quantity of money.
Despite the decline in the value of money, the model predicts full acceptance of money even when this is supplied in large quantities.

Moving the attention on the effects of inflation, fig. 1a suggests that type 2 individuals do not accept fiat money for good 1 if the inflation is sufficiently high. It also says that type 1 agents may switch from speculative to fundamental strategies as inflation goes up. Indeed, when $\delta_{m}$ is high, individuals holding commodities are likely to wait longer to acquire their consumption goods, forgoing the liquidity benefits of money. Conversely, for low levels of inflation, the liquidity benefits outweigh the cost of seigniorage. Fig. 3b plots dependence of $\Delta_{2,3}^{1}, \Delta_{m, 1}^{2}$, and $\Delta_{m, 1}^{3}$, from the inflation tax, $\delta_{m}$, when $Q=1 / 3$. The results indicate that the value of money (relative to good 1) for individuals of type 2 and type 3 decreases when $\delta_{m}$ goes up. At intermediate levels of inflation, multiple equilibria exist. However, in the lower and higher extremities of the interval [ $0,0.09$ ], respectively, the speculative and fundamental equilibria are unique. The increase in $\delta_{m}$ induces a shift in behavior from speculative to fundamental among type 1 individuals because the decline in the value of fiat money leads type 2 individuals to prefer good 1 to fiat money in the upper range of the interval $[0,0.09]$. Consequently, type 2's holdings of good 1 increase relative to those of type 3, rendering type 1's fundamental strategy more profitable than the speculative one. Based on these observations, we propose two additional hypotheses: Hypothesis 3 (H3): Speculative behavior decreases as inflation rises.
Hypothesis 4 (H4): The acceptability of money decreases with inflation.
As already noted, when the rate of inflation is high for type 1 individuals speculative strategies are not optimal, because they have a higher chance in a paring with agents 2 to obtain their consumption good directly. Therefore, the prediction of H3 hinges on the occurrence of H 4 . To test H 3 , we evaluate whether the frequency at which type 1 agents trade good 2 for good 3 decreases with the inflation tax. And we test H4 by considering whether the frequency at which type 2 and type 3 agents and trade good 1 for money decreases with the inflation rate.

### 2.4 Inflation and Welfare

The objective of this section is to explore the welfare effects of monetary policy. In our set-up, the inflation tax gives an incentive to the money holder to pass it on to someone else. However, money also provides a liquidity benefit for the holder, as it reduces the expected waiting time for consumption. Provided the inflation rate does not alter the type of equilibrium, its losses are simply due to the production cost $D$ and the difference between the value of holding money and holding a commodity. The welfare of a type $i$ agents, $W_{i}$, and the average welfare, $W$, can be calculated as $W_{i}=$ $\left(p_{i, i+1} V_{i, i+1}+p_{i, i+2} V_{i, i+2}+p_{i, m} V_{i, m}\right) / \theta_{i}$ and $W=\sum_{i=1}^{3} \theta_{i} W_{i}$ respectively. Similarly, the consumption rate of good $i$ is $C_{i}=\sum_{k} \sum_{j} p_{i, j} p_{k, i} s_{i, j}^{k}$ and the average consumption rate is $C=\sum_{i=1}^{3} \theta_{i} C_{i}$. Fig. 4a shows that within each equilibrium, inflation tax reduces both $W_{i}$ and $W$. We find that the welfare losses due to inflation are modest if inflation
does not alter economy's type of equilibrium. This result echos that of Lucas (2000). ${ }^{5}$ Nevertheless, fig. 4a shows that if the inflation spike is large enough to cause the economy to move from the speculative to the fundamental equilibrium - for instance it goes from 0 to 8 percent - average welfare, $W$, drops by about one-third (see table 2). A large share of this decline is explained by the lower production and consumption activity in the fundamental as compared to the speculative equilibrium. Production and consumption decline by $22 \%$ (see table 2). The changes in welfare associated with inflation depend on the specific position of individuals with respect to consumption, trade, and production (a similar point is made, among others, in works by Albanesi 2007, Doepke and Schneider 2006, Coibion et al. 2017, and Chiu and Molico 2010). The largest welfare drop, is that of type 2 agents, $-56.8 \%$. The percentage welfare decline of type 1 and type 3 agents is 20 and 27 percent, respectively. The ratio $W_{2} / W_{1}$ drops from slightly above 1 in the speculative region to around 0.55 in the fundamental region (fig. 4b). Indeed, the change of strategies of type 1 from speculative to fundamental penalizes more strongly type 2 agents, as these are the producers of good 3. When the stock of money drives the economy from the speculative to the fundamental equilibrium, there is a similar decline in aggregate welfare (approximately one-third). Also in this case, type 2 individuals are hit the hardest, experiencing a welfare decline of around $50 \%$.

## 3 Small Number of Agents

Up to this point we derived Nash equilibria in an economy populated by a continuum of agents. The properties of equilibria in an environment with a continuum of agents, however, are not necessarily applicable in situations with a finite number of agents, as pointed out by Judd (1985), Feldman and Gilles (1985), and Khan et al. (2020). To address this concern, we adapt the analysis of the baseline economy to an economy populated by 18 agents, which is the number of subjects in the laboratory experiments (see Online Appendix A.2). We then use the framework to compute whether, under the same set of parameters reported in table 1, the distribution of commodities and money implied by the four Nash strategies of fig. 1a (fundamental with full acceptability of money, fundamental with partial acceptability of money, speculative with full acceptability of money, and speculative with partial acceptability of money) still holds for an economy populated by a small number (18) of agents. ${ }^{6}$ Similar to fig. 1a, fig. 1b partitions the $\left(\delta_{m}, Q\right)$ space of the 18 -agent economy into Nash equilibria regions. The comparison

[^4]between figs. 1a and 1 b shows that the regions of equilibria in the two economies are alike. In anticipation of the laboratory experiments, it is instructive to compare agents' payoffs in the two economies on specific points within the ( $\delta_{m}, Q$ ) space. Tables 3 and 4 report the value functions of six such points. Specifically, we computed the value function $V_{i, j}(X, \mathbf{s})$ associated with strategy $\mathbf{s}$, for a state of the economy $X=\left(x_{1}, x_{2}, \ldots, x_{N}\right)$, where $x_{a}$ represents the type of good held by an agent $a$. The value reported tables 3 and 4 are averages of at least $10^{6}$ simulated series, each of which consists of 100 trading rounds (because of discounting, making the series longer would have a negligible effect the value of $\left.V_{i, j}(X, \mathbf{s})\right)$. The initial distribution of goods and money in each treatment approximates the steady state equilibrium of the economy with a continuum of agents discussed in section 2, under the quantity of money and inflation associated with that particular treatment. These initial distributions are given in table 6. In what follows we discuss the most salient results.
No inflation. Section 2.4 established that in the baseline economy with no inflation tax, the speculative strategy is less appealing for type 1 agents as the stock of money increases. This result equally holds in the 18 -agent economy. Tables 3b and 3c compare changes in $\Delta_{2,3}^{1}$ in the two economies when moving between equilibria that differ for the stock of money. For example, when the stock of money goes from low $\left(L_{0}\right)$ to medium $\left(M_{0}\right), \Delta_{2,3}^{1}$ increases by 8.58 in the baseline economy, and by 1.33 in the 18 -agent economy. This demonstrates that in both economies the fundamental strategy becomes more valuable relative to the speculative one (see table 4c). Additionally, tables 4 a and 4 b demonstrate that when the money supply changes from low to medium or low to high the relative value of money compared to that of good 1 decreases both for type 2 and type 3 agents in either economy.
Inflation. In the baseline economy, inflation incentivizes fundamental behavior, as demonstrated in table 3b. Inflation increases the difference in payoffs, $\Delta_{2,3}^{1}$, between the fundamental and speculative strategies, as shown by all three scenarios of low, medium, and high money supply. For the 18 -agent economy, the value of $\Delta_{2,3}^{1}$ cannot be calculated for all treatments of interests. However, in a pair-wise treatment comparison of interest for which we could obtain data, that is, $L_{0}$ vs. $L_{+}$the results is similar to that obtained for the infinite economy: inflation weakens the speculative incentives.

Regarding the acceptability of money, a comparison of the last two columns of tables 3 c and 4 c indicates that the inflation tax devalues fiat money relative to good 1 in a similar way in the two economies. For instance, when $Q=M, \Delta_{m, 1}^{2}$ is equal to - 18.8 in the baseline economy and to -15.73 in the 18 -agent economy. The values of $\Delta_{m, 1}^{3}$ are -9.09 and -8.79 respectively. Overall, money and inflation appear to have similar effects in the two economies.

### 3.1 Error Prone Agents

The previous section assumed that subjects in the 18-agent economy followed Nash strategies. This section relaxes that assumption and allows individuals to make "errors" in trade decisions that do not involve their consumption goods. These errors may stem from the individuals' limited capacity or attention when processing complex information that push
them away from the best decision. This characterization of agents' interaction may capture human behavior in laboratory experiments more accurately. Accordingly, we assume that individuals follow either a Nash strategy, with probability $(1-q)$, or the opposite of that Nash strategy with probability $q$ :

$$
\begin{equation*}
\mathbf{s}^{E}(q)=(1-q) \mathbf{s}+q(\mathbf{1}-\mathbf{s}), \tag{5}
\end{equation*}
$$

where $\mathbf{1}_{i, j}=1$ for $i, j=1,2,3$.
Table 5a shows the outcome of numerical experiments with $q=0.1$. The values $V_{i, j}(X, \mathbf{s})$ refer to the same initial value of $X$ used in the previous section (see table 6). Despite the noise associated with the error, the values of $\Delta_{j, k}^{i}$ - when available - are largely consistent with our calculations based on Nash strategies. For instance, when the quantity of money is low, there is speculative behavior with full acceptance of the money both with or without inflation (table 5b). Furthermore, the error does not alter the key insight that inflation reduces the acceptability of money (table 5c). These results are consistent with the baseline model and the error-free 18-agent model (see tables 3c and 4 c$)$. In agreement with the error-free 18-agent economy and the baseline economy, inflation reduces the desirability of speculative choices.

## 4 Laboratory Experiments

### 4.1 Experimental Design

We conducted 15 sessions of human subject experiments to test hypotheses derived from our model. Each session involved five treatments, each with the same parameters for consumption utility, production cost, and cost of holding goods, as shown in table 1. The treatments varied by the quantity of fiat money $(Q)$ and inflation $\operatorname{tax}\left(\delta_{m}\right)$. In treatments $L_{0}, M_{0}$, and $H_{0}, \delta_{m}$ was set to 0 , and the fraction of the population storing good $m$ was $2 / 18,6 / 18$, and $10 / 18$, respectively. In the other two treatments, $M_{+}$and $H_{+}, \delta_{m}$ was set to 0.08 and $Q$ was $6 / 18$ and $10 / 18$, respectively. We performed the 15 sessions at the Experimental Economics Laboratory of the Institute of Social and Economic Research (ISER) at Osaka University between January and July 2020. Ten sessions took place in January and February 2020, and five sessions took place in July 2020. The laboratory closed due to COVID-19 for several weeks after February 2020. We recruited a group of 18 subjects for each session, with a total of 270 subjects recruited online through the ORSEE system. Out of these subjects, 77 were female, and their average age was 22 as they were students from Osaka University. Each subject participated in five games, each associated with one of the five treatments described above. Although we recruited participants for two-hour sessions, due to the random termination rule, sessions lasted an average of 100 minutes, including instruction and quiz time. No subject participated in more than one session, and none had prior experience with the type of games played in the laboratory.

The Online Appendix B provides an English translation of the written instructions given to participants at the beginning of each session. We also played a pre-recorded
audio file of the written instructions to reinforce the content. The instructions informed participants that their type may change from one game to the next but would remain the same throughout each game. They also detailed the rules of the experiment, the objectives of each player type, and how to earn and lose points based on the values reported in table 1. Participants learned that acquiring the consumption good would give them 130 points, that production would cost 30 points, and that carrying a commodity would depend on his or her type, according to table 1. The instructions also explained that holding a "token", which was the name we used for fiat money, would cost no points but could be confiscated if displayed on their screen. Participants were informed that nobody could immediately earn points by acquiring a token as it cannot be consumed.

After answering any questions about the rules of the game, participants took a comprehension quiz to assess their understanding of the experimental instructions. The quiz asked participants to answer the same question until they selected the correct answer. Participants then learned about the computer interface.

For this experiment, including the comprehension quiz, we used the z-Tree computer program (Fischbacher, 2007). The program randomly matches participants, informs them of relevant information about their trading decisions, keeps track of historical information such as goods in storage, trading decisions, and accumulated points, and calculates summary statistics to give insight into the historical distribution of strategies.

Each game consists of one or more sets of ten trading rounds. The interval during which a trading round takes place is referred to as a period. At the start of each period, the computer program randomly matches all participants into pairs. Participants can then see on their computer screen information about the good held by their paired counterpart (refer to fig. 7 in the Online Appendix B). The player's screen also displays information about the player's own type, the type of good in storage, the cost of holding any of the three types of commodities, and the probability of fiat money confiscation. In addition, the screen reminds players about the payoffs, including a gain of 130 points for acquiring their consumption goods, a deduction for the storage cost based on the good being stored, and a cost of 30 points due to the production of a new good. Each player starts the first round with an endowment of 150 points to prevent the cumulative points from turning negative during the game. Throughout the game, players accumulate points that eventually are converted into cash. The end of this section describes in further details this conversion.

Each game is associated with one of five treatments, and the order in which the treatments are applied is different from session to session. The treatments are organized so that over the course of 15 sessions, each treatment takes up one of five possible positions three times. Each player starts the first round of the game with one unit of commodity or one token. Following a similar approach to that used in our numerical exercise with an 18-agent economy, the software randomly chooses the type of good or token for each player to reproduce the distribution of commodities and fiat money associated with a particular treatment (see table 6). When two players are paired, they are asked if they want to trade the goods in their inventories. Players can only respond with Yes or No. They cannot trade again within the same period. If both players agree to trade, the goods are exchanged on a one-to-one basis. Before making a decision, a player can view the following information on the screen:
a) the probability of token confiscation
b) the probability that the game will end at the end of the period, if it was still running up to that point.
c) the number of agents of type $i$ holding good $j$. By selecting the "History" tag player could also switch from the current distribution of holdings to the historical averages, counting from the beginning of the game. A "Current" tag would bring the screen back to the current distribution.

After the player makes a trade decision (Yes or No), a new screen appears (see fig. 8), displaying the player's holding and points, as well as the governmnent's confiscation of tokens (if any) during the current period. At the end of each 10-round block, the screen shows whether the game will continue for another 10 periods.

The game is randomly terminated following a common practice in such experiments. The game has a constant probability of ending (0.1) in any period, determined by a random number generated by the computer. Specifically, following the completion of a trading round, the computer picks a number from a uniform distribution over $[0,1]$. If this number is between 0 and 0.1 the current trading round is the last one of the game considered for the calculation of the players' payoffs. Nevertheless, the game continues until the 10th period, at the end of which players learn that the game is over. The information delay allows for an increase in the number of observations (see Fréchette and Yuksel (2017) for a discussion). There is no restriction on the number of 10-period blocks that can be played in a single game, with the longest games lasting 40 periods.

While we recruited subjects for sessions of 120 minutes we did not tell them in advance how many games they would play. Of the 75 total games ( 5 per 15 sessions), 42 were over at the 10th period, 22 at the 20th, 7 at the 30 th, and 4 at the 40 th period. The distribution of periods across sessions is: 50 (1), $60(1), 70(6), 80(1), 90(1), 100(3), 110(1), 120(1)$, where the figures in parenthesis indicate the number of sessions. Finally, to decide the reward, once the last game of the session has been played, we asked the computer program to choose one game at random among all the five played in that session. The basis for the cash payment was the cumulative points recorded by each player at the end of the randomly selected game. We converted one point into 10 JPY. Each participant received an additional reward of 500 JPY . We communicated the reward scheme at the beginning of each session, along with the other instructions about the rules of the experiment. A participant earned an average of 2910 JPY (approximately $\$ 27$ ).

### 4.2 Experimental Results

This section presents an overview of the results of our experimental sessions focusing on the aggregate behavior of players of a given type. Next section studies the data through statistical analysis. Table 7 reports the frequencies at which the three types of individuals were willing to trade away the good in their holdings for a different good or for fiat money, when such a circumstance materialized. These frequencies are obtained by studying 11070 matches, observed over the 15 sessions.

Consumption Good. In line with the model's assumption that individuals acquire their consumption goods whenever such opportunities arise, individuals tended to trade their goods or fiat money for their consumption goods. For example, type 1 offered fiat money or goods 2 or 3 for good 1 at a frequency between $97 \%$ and $100 \%$ across the five treatments. This outcome is not surprising and indicates that most subjects understood the basic structure of the payoff and that this motivated them to play the game.
Money quantity and speculative behavior (Hypothesis 1). The model predicts that, in absence of an inflation tax, speculative strategies are more likely to occur when the stock of money, $Q$, is low. Conversely, a high $Q$ would lead to a greater likelihood of fundamental strategies being observed. Table 7 reveals that in treatments $L_{0}, M_{0}$, and $H_{0}$, type 1 tends to trade good 2 for good 3 at frequencies below $50 \%$. This suggests that fundamental strategies are more likely to be observed, regardless of the level of $Q$. However, the table also shows that the frequency of speculative behavior is lower in $M_{0}$ and $H_{0}$ compared to $L_{0}$, indicating that the availability of fiat money may discourage some individuals from pursuing speculative strategies. The bottom section of table 7 reports the trade of good 3 for good 2, and it shows that speculative behavior is prevalent in treatment $L_{0}$, with a frequency of 0.71 (calculated as $1-0.29$ ). However, this behavior is not present in treatments $M_{0}$ or $H_{0}$. Thus, it seems that the trade of good 3 for good 2 aligns more closely with the model's prediction shown in fig. 1a, compared to the trade of good 2 for good 3 .
Acceptability of fiat money (Hypothesis 2). Participants showed a strong preference for accepting fiat money in the treatments without the inflation $\operatorname{tax}\left(L_{0}, M_{0}\right.$, and $\left.H_{0}\right)$. Type 1 subjects, for instance, frequently offer good 2 in exchange for fiat money, with a frequency range of $77 \%$ to $84 \%$. Type 2 and type 3 subjects also have similar tendencies, with frequency ranges of $79 \%$ to $95 \%$ and $72 \%$ to $89 \%$ respectively, as shown in the table 7 panels I-III. Another indication that individuals had a strong preference for fiat money is the low frequency at which they use fiat money to acquire a good other than their consumption good. For instance, type 1 subjects engage in this behavior in only $12 \%$ of cases. These findings confirm the existence of monetary equilibria, even with decreased storage costs for goods 2 and 3 compared to those used in the experiment by Duffy and Ochs (2002) (as seen in their table 5). Despite a significant decrease in storage costs, fiat money continues to be widely accepted.
Inflation and speculative trading behavior (Hypothesis 3). The frequency distributions in the raw experimental data reveal no evidence of the inflation tax affecting the buying and selling decisions of type 1 study participants. The behavior of type 1 subjects in trading good 2 for good 3 remained consistent across treatments $M_{0}$ and $M_{+}$, as shown in the middle and bottom sections of table 7 panel I. This observation contradicts the model's prediction that inflation would decrease speculative behavior for a medium stock of money $(Q=M)$.

Inflation and the acceptability of fiat money (Hypothesis 4). The study participants' acceptance of money clearly showed sensitivity to the inflation tax. As inflation increased, the frequency of exchanging fiat money for commodities, other than the consumption good, rose for all three types of agents, while the frequency of exchanging
commodities for fiat money decreased. This can be observed by comparing the frequencies in treatments $M_{0}$ to those in treatments $M_{+}$and in treatments $H_{0}$ to those in treatments $H_{+}$.

## 5 Statistical Analysis

In this section we analyze experimental data. One important challenge is to account for the variability in the choices of individuals of the same type under similar trade situations. The literature offers various methods for addressing this issue (for a review, see Moffatt 2016). Our approach is to first analyze the experimental data using a simple statistical model that closely captures the behavioral assumptions of the theoretical environment. We then extend the model to explore heterogeneity among agents. Suppose that participants are all alike, in the sense that in similar circumstance act in the same way, according to incentives of their personal payoffs. Nevertheless, individuals can also make mistakes and choose the opposite strategy of what would be the optimal one. One objective of the statistical analysis is to estimate the probability that individuals adopt a given trading strategies, and how such probability changes with inflation and with the quantity of money. We then turn the statistical analysis to test the four predictions of the model outlined in section 2.4 (H1-H4). We use the following notation in this section. Let $s_{1}(i, T)$ denote the likelihood that an individual of type $i$ trades good $i+1$ for money, $m$, and $s_{2}(i, T)$ indicate the likelihood that a type $i$ individual trades good $i+2$ for $m$ in treatment $T$. Let $s_{3}(i, T)$ signify the likelihood that a type $i$ individual trades $i+1$ for $i+2$ in treatment $T$. We use $N_{1}(i, T)$ to symbolize the number of opportunities type $i$ individuals had in treatment $T$ to trade good $i+1$ for money, and $n_{1}(i, T)$ to stand for the number of times they chose to trade. In this situation, $n_{1}(i, T)$ is described by a binomial distribution with parameters $N_{1}(i, T)$ and $s_{1}(i, T)$, that is

$$
\begin{equation*}
\mathbb{P}\left(n_{1}(i, T)=n \mid N_{1}(i, T)=N\right)=\binom{N}{n} s_{1}(i, T)^{n}\left(1-s_{1}(i, T)\right)^{N-n} \tag{6}
\end{equation*}
$$

The Method of Moments (MM) estimator for $s_{1}(i, T)$ is calculated as $\tilde{s}_{1}(i, T)=$ $n_{1}(i, T) / N_{1}(i, T)$ (see Chapter 7.2 of Devore, 2010). Based on this, speculation level in treatment $T$ can be calculated as

$$
\begin{equation*}
\tilde{s}_{1}(3, T)=n_{1}(3, T) / N_{1}(3, T) \tag{7}
\end{equation*}
$$

Similar equations can be derived for the MM estimators of $s_{2}(i, T)$ and $s_{3}(i, T)$. Table 8 lists the estimators' values, along with their confidence intervals, for the five laboratory treatments $L_{0}, M_{0}, H_{0}, M_{+}$, and $H_{+}$. In line with the summary statistics displayed in table 7 , the probability of accepting money, as shown in $\tilde{s}_{1}(i, T)$ and $\tilde{s}_{2}(i, T)$, is generally high for all types and often exceeds $50 \%$. However, type 1 individuals select speculative strategies with a probability $\tilde{s}_{3}(1, T)$ that tend to be below $50 \%$. By comparing the trading probabilities across treatments, we gain insight into how individuals respond to the inflation tax and the quantity of money in the system. Table 8 reveals that the
quantity of money does not seem to alter the acceptance of money in the treatments without an inflation tax, $L_{0}, M_{0}, H_{0}$. Nonetheless, as we move from $L_{0}$ to $M_{0}$ or from $L_{0}$ to $H_{0}$, we observe a decrease in the probability of type 1 individuals engaging in speculative strategies, $\tilde{s}_{3}(1, T)$. But this probability increases when moving from $M_{0}$ to $H_{0}$.

To further examine the differences in data from various treatments, we perform a series of hypothesis tests for the statements H1-H4 outlined in section 2.4. H1 posits that type 1 individuals are less likely to engage in speculative behavior with an increase in the quantity of fiat money. This means that type 1 individuals are less likely to speculate in $T=M_{0}$ compared to $T=L_{0}$, or $s_{3}\left(1, M_{0}\right)<s_{3}\left(1, L_{0}\right)$. To test this, we establish a null hypothesis of $h_{0}: s_{3}\left(1, M_{0}\right) \geq s_{3}\left(1, L_{0}\right)$ against the alternative hypothesis $h_{a}: s_{3}\left(1, M_{0}\right)<s_{3}\left(1, L_{0}\right)$. Rejection of $h_{0}$ supports H1, and a low $p$-value indicates a high level of statistical confidence in the rejection of $h_{0}$. We calculate $p$-values using the large sample tests for the statistics.

$$
\begin{equation*}
z=\frac{\tilde{s}_{3}\left(1, M_{0}\right)-\tilde{s}_{3}\left(1, L_{0}\right)}{\sqrt{\tilde{s}(1-\tilde{s})\left(\frac{1}{N_{3}\left(1, M_{0}\right)}+\frac{1}{N_{3}\left(1, L_{0}\right)}\right)}}, \tag{8}
\end{equation*}
$$

where

$$
\tilde{s}=\tilde{s}_{3}\left(1, M_{0}\right) \frac{N_{3}\left(1, M_{0}\right)}{N_{3}\left(1, M_{0}\right)+N_{3}\left(1, L_{0}\right)}+\tilde{s}_{3}\left(1, L_{0}\right) \frac{N_{3}\left(1, L_{0}\right)}{N_{3}\left(1, M_{0}\right)+N_{3}\left(1, L_{0}\right)} .
$$

The distribution of $z$ is approximated with a normal distribution because of the Central Limit Theorem (see, among others, Devore 2010, Chapter 9.4).

The results of these tests are reported in table 9. The null hypothesis $h_{0}$ that $s_{3}\left(1, M_{0}\right) \geq s_{3}\left(1, L_{0}\right)$ received a small $p$-value of 0.001 , which supports H1's statement that type 1 speculative behavior decreases with the quantity of fiat money. However, the results are not as clear-cut when comparing treatments with differing amounts of money, keeping the same inflation rate. For instance, the $p$-values obtained in the comparisons between $H_{0}$ and $L_{0}$, and $H_{0}$ and $M_{0}$ are high, at 0.181 and 0.898 respectively. Conversely, the comparison between $H_{+}$and $L_{+}$resulted in a small $p$-value of 0.005 . In conclusion, the results summarized in table 9 suggest that fiat money reduces type 1 individuals' inclination towards speculation in two out of four treatment comparisons.

We studied the consequences of the quantity of money on the acceptance of money by type 2 and type 3 agents by testing their behavior when trading good 1 , which has the lowest storage cost, for money. Our results tend to support H2's statement that money acceptance decreases with the quantity of money, as we rejected the null hypothesis in three out of four pairwise treatment comparisons (i.e., $L_{0}$ vs $H_{0}, M_{0}$ vs $H_{0}$, and $M_{+}$ vs $H_{+}$) with $p$-values between 0.001 and 0.073 . However, in the $L_{0}$ vs $M_{0}$ comparison, the $p$-value of 0.887 showed no significant difference in money acceptance, in fact, the money acceptance difference had the opposite sign from what we hypothesized. Type 3 agents showed little responsiveness to the quantity of money, with only a low $p$-value of 0.050 observed in the $L_{0}$ vs $M_{0}$ comparison. There was no evidence in the other three
comparisons ( $L_{0}$ vs $H_{0}, M_{0}$ vs $H_{0}$, and $M_{+}$vs $H_{+}$) that the stock of money in circulation influenced type 3's evaluation of fiat money, with two out of three estimates carrying an opposite sign

The hypothesis tests for the H3 statement that inflation reduces type-1 speculative behavior produce inconsistent results. The $p$-value for the comparison between $H_{0}$ and $H_{+}$is 0.05 , but the $p$-value for the comparison between $M_{0}$ and $M_{+}$is 0.921 , which is very high.

Finally, the experimental data firmly supports the H4 statement that inflation decreases individuals' appraisal of money. In both type 2 and type 3 agents, small $p$ values were obtained when comparing their acceptance of fiat money in inflating and non-inflating scenarios (as shown in the bottom section of table 9).

## 6 Robustness of the Statistical Analysis

In line with the economic model environment, the statistical analysis in the previous section assumes that individuals have the same decision-making process regarding trading strategies. However, in experimental studies it is common to account for heterogeneity in people's choices. People may differ in their ability to maximize payoffs, perceive time-horizons, or willingness to take the risk of a speculative trade. While individual characteristics such as decision-making abilities and perception of time-horizon cannot be directly observed, a closer examination of experimental data may provide insight into the possibility of refining results with further statistical analysis.

Fig. 5a illustrates the decisions made by 43 type 1 agents in treatment $M_{0}$. These agents had to choose between goods 2 and 3 in multiple periods. Out of the 43 individuals, 19 consistently favored good 2,8 consistently favored good 3 , and the remaining 15 individuals demonstrated ambiguity in their choices, favoring one good over the other half of the time. Similarly, Fig. 5b displays a bimodal distribution for type 2 agents in their choice between good 1 and fiat money in treatment $H_{+}$. These observations suggest that even agents of the same type in similar circumstances may take opposing trading decisions. Next, we then present a statistical model that allows for two individuals of the same type to take opposite strategies in similar circumstances.

### 6.1 A Mix of Two Binomial Distributions

Suppose at the start of the game, a fraction $q_{1}(T)$ of type 1 individuals in treatment $T$ choose a speculative strategy, while the remaining $1-q_{1}(T)$ choose a fundamental strategy. These individuals may deviate from their pre-set strategies with a probability of $p_{1}(T)$. These deviations could result from either random mistakes or strategic decisions made based on information about the economy that participants can view on their screens.

Denote the number of opportunities for type 1 individual $a$ to participate in an exchange between goods 2 and 3 in treatment $T$ as $N_{1}(a, T)$. Let $n_{1}(a, T)$ be the number of times this individual chooses good 3 over good 2 . The distribution of $n_{1}(a, T)$, given
$N_{1}(a, T)$, can be expressed as a combination of two binomial distributions:

$$
\begin{align*}
& \mathbb{P}\left(n_{1}(a, T)=n, \mid, N_{1}(a, T)=N\right)=  \tag{9}\\
& \quad q_{1}(T)\binom{N}{n} p_{1}(T)^{n}\left(1-p_{1}(T)\right)^{N-n}+\left(1-q_{1}(T)\right)\binom{N}{n} p_{1}(T)^{N-n}\left(1-p_{1}(T)\right)^{n}
\end{align*}
$$

We use Generalized Method of Moments (GMM) estimators, $\hat{p}_{1}(T)$ and $\hat{q}_{1}(T)$, to compute the probability that type 1 agents choose a speculative strategy. This probability is expressed as

$$
\begin{equation*}
\tilde{s}_{1}(3, T)=\tilde{p}_{1}(T) \tilde{q}_{1}(T)+\left(1-\tilde{p}_{1}(T)\right)\left(1-\tilde{q}_{1}(T)\right) \tag{10}
\end{equation*}
$$

In a similar fashion, we calculated the probabilities of money acceptance with regards to good 1 for both type 2 agents $\left(\hat{s}_{2}(2, T)\right)$ and type 3 agents $\left(\hat{s}_{3}(3, T)\right)$.

Table 10 compares these estimates to those obtained from the baseline statistical analysis, $\hat{s}_{3}(1, T), \hat{s}_{2}(2, T)$, and $\hat{s}_{1}(3, T)$. The estimates of speculative behavior and money acceptance, calculated with the mixture of two binomial distributions and reported in table 10, are comparable to those obtained through the basic statistical analysis (table 8). Table 10 reveals that the speculative probability in $L_{0}$, calculated using the two binomial distribution functions, is 0.35 , which is close to the 0.41 calculated from the baseline statistical model. Nevertheless, calculating the probability of a strategy as an average of probabilities offers a deeper understanding of how people made their decisions. As shown in table 10, the speculative probability of 0.36 is composed of an estimated 0.27 fraction of people who chose a speculative strategy and a $17 \%$ deviation from the original strategy, whether it was the fundamental or speculative strategy.

### 6.2 Normal Random Variable

A different way to take into account heterogeneity among participants is to incorporate it through a continuous variable. Suppose that a type 1 individual $a$ in treatment $T$, decides on a trade of good 2 for good 3 by comparing the outcome of a normally distributed noise, $\epsilon$, with a personal threshold $\Gamma_{1}(a, T)$. The noises $\epsilon$ are independent among agents and time periods. The personal threshold is given by $\Gamma_{1}(T, a)=D_{1}(T)+b_{1}(a)$, where $D_{1}(T)$ represents type 1's tendency to speculate in treatment $T$ and $b_{1}(a)$ captures individual $a$ 's bias towards or against the speculative strategy. Individual $a$ then has a probability of $\Phi\left(D_{1}(T)+b_{1}(a)\right)$, of preferring good 3 over good 2 , where $\Phi(x)$ is the cumulative distribution function of a standard normal random variable. Due to the limited size of our data set, we cannot calculate $b_{1}(a)$ individually. So, we assume that $b_{1}(a)$ follows a normal distribution with a mean of zero and a variance of $v_{1}$. The parameter $D_{1}$ accounts for any deviation of $b_{1}(a)$ from its mean.

To estimate the average fraction of speculative choices for treatment $T$, we calculate $\bar{s}_{1}(3, T)$ as follows:

$$
\begin{equation*}
\bar{s}_{1}(3, T)=\int_{-\infty}^{\infty} \Phi\left(D_{1}(T)+b_{1}\right) \frac{1}{\sqrt{2 \pi v_{1}}} e^{-b_{1}^{2} / 2 v_{1}} d b_{1} \tag{11}
\end{equation*}
$$

Similar probability functions describe the choices between fiat money and good 1 for type 2 and type 3 agents. These functions have parameters $\left(D_{2}(T), b_{2}(a)\right)$ and $\left(D_{3}(T), b_{3}(a)\right)$, where $b_{2}(a)$ and $b_{3}(a)$ are normally distributed with zero mean and variances of $v_{2}$ and $v_{3}$, respectively. By using these probability functions, we estimate $s_{2}(2, T)$ and $s_{3}(1, T) .{ }^{7}$

Table 11 presents the results of our statistical analysis for the estimates $s_{1}(3, T)$, $s_{2}(2, T)$, and $s_{3}(1, T)$. It shows that the fraction of type one individuals who speculate, $\bar{s}_{1}$, decreases from 0.35 to 0.28 when moving from $L_{0}$ to $M_{0}$, and from 0.31 to 0.17 when moving from $M_{+}$to $H_{+}$. Conversely, the fraction increases from 0.28 to 0.31 when moving from $M_{0}$ to $H_{0}$. The pattern and size of these changes are in line with the findings of the previous two statistical methods, which are also summarized in the table for convenience. Table 11 also indicates that the estimates of money acceptance, $\bar{s}_{2} \bar{s}_{1}$, are in line, and somewhat more marked, than what obtained with the other two statistical methods. We also find that the probit approach yields $p$-values for the null hypotheses similar to those shown in table 9 for the baseline statistical analysis. We also analyzed the experimental data using the random effects probit model, as for instance in Duffy and Puzzello (2022), and the multilevel model (see Moffatt, 2016). However, both approaches led to the same conclusions as our simpler statistical models.

## 7 Conclusion

Evaluating the effects of monetary policy is a challenging task that requires observing how individuals respond to it and articulating how microeconomic mechanisms are transmitted to the macroeconomy. In this study, we examined the consequences of a change in the quantity of money and inflation in a KW search model and tested the predictions in laboratory experiments.

One advantage of our setup was the ability to use fiat money and inflation as instruments to induce the economy to move between equilibria with different levels of money acceptance, aggregate productivity, and welfare. We found that an increase in the inflation rate caused only modest changes in welfare, comparable to those calculated by Lucas (2000), if the economy remained in the same equilibrium. However, if inflation caused individuals to coordinate on a different equilibrium, the welfare effects were significant. These findings imply that reducing inflation could be a costly affair or could have only small welfare consequences, depending on whether agents coordinate on a new equilibrium or maintain the pre-policy trading strategies. The model also suggested that inflation may reduce people's confidence in fiat money, potentially leading to a reduction in trading opportunities. Our laboratory experiments provided clear support for the hypothesis that inflation has an adverse effect on people's trust in money, but it did not support the notion that it alters people's expectations to the point of changing their trading strategies. In other words, we did not observe evidence that inflation could move the economy from a fundamental to a speculative equilibrium or vice versa. Therefore, the inflation welfare cost in the KW environment is close to Lucas (2000)'s estimate.

[^5]As the experimental data were generated in a small-scale environment, we adapted the original model, which assumes a continuum of agents, to a model with a finite number of agents. This extension allowed us to compare the equilibria of the two types of economic models. While the key insights into people's behavior emerged from a simple reading of the experimental data, we also proposed statistical models to interpret the variations in the data. The baseline statistical model assumed that people follow the same decision process under similar circumstances. We then investigated how the results would change if we allowed for some heterogeneity in people's characteristics. Despite a substantial degree of heterogeneity in people's choices, the main conclusion of the baseline statistical model remained unchanged.

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## Tables

Table 1: Parameters

| Disc. Rate | Utility |  |  | Storage costs |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\rho$ | $U$ | $D$ | $u$ | $c_{1}$ | $c_{2}$ | $c_{3}$ |
| 0.1 | 130 | 30 | 100 | 4 | 10 | 20 |

Table 2: Comparison between two Equilibria
(a) Consumption
(b) Welfare

|  | $C_{1}$ | $C_{2}$ | $C_{3}$ | $C$ |
| :---: | :---: | :---: | :---: | :---: |
| Spec | 0.059 | 0.059 | 0.060 | 0.059 |
| Fund | 0.047 | 0.046 | 0.046 | 0.046 |
| $\%$ | -20.3 | -22.0 | -23.3 | -22.0 |


|  | $W_{1}$ | $W_{2}$ | $W_{3}$ | $W$ |
| :---: | :---: | :---: | :---: | :---: |
| Spec | 83.55 | 85.71 | 148.32 | 105.84 |
| Fund | 66.45 | 36.99 | 107.88 | 70.44 |
| $\%$ | -20.4 | -56.8 | -27.2 | -33.4 |

Note: In the speculative equilibrium $\delta_{m}=0$; in the fundamental equilibrium $\delta_{m}=0.08 ; Q=1 / 3$. The third line reports percentage differences from the speculative to the fundamental equilibrium.

Table 3: Baseline Economy (Continuum of Agents)
(a) Value Functions

| $\left(\delta_{m}, Q\right)$ | $V_{1,2}$ | $V_{1,3}$ | $V_{1, m}$ | $V_{2,3}$ | $V_{2,1}$ | $V_{2, m}$ | $V_{3,1}$ | $V_{3,2}$ | $V_{3, m}$ | Equilibrium |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $L_{0}$ | 93.2 | 99.8 | 164 | 103 | 129 | 142 | 182 | 145 | 194 | Spec-F |
| $M_{0}$ | 68.6 | 72.5 | 132 | 74.3 | 102 | 114 | 148 | 118 | 159 | Spec-F |
| $H_{0}$ | 32.3 | 31.9 | 87.3 | -6.19 | 52.3 | 62.3 | 86.7 | 72 | 98.1 | Fund-F |
| $L_{+}$ | 94.7 | 95 | 130 | 98.1 | 125 | 123 | 177 | 142 | 180 | Spec-P |
| $M_{+}$ | 58.5 | 47.8 | 81.9 | 0.432 | 62.6 | 54.8 | 102 | 88.3 | 105 | Fund-P |
| $H_{+}$ | 35 | 15.1 | 46.1 | -12.7 | 37.6 | 29.4 | 70.4 | 58.1 | 71.9 | Fund-P |

(b) Differences of Value Functions

| $\left(\delta_{m}, Q\right)$ | $\Delta_{2,3}^{1}$ | $\Delta_{m, 1}^{2}$ | $\Delta_{m, 1}^{3}$ |
| :---: | :---: | :---: | :---: |
| $L_{0}$ | -12.4 | 12.4 | 12.4 |
| $M_{0}$ | -3.92 | 11.0 | 11.6 |
| $H_{0}$ | 0.45 | 10.0 | 11.3 |
| $L_{+}$ | -0.40 | -1.56 | 3.10 |
| $M_{+}$ | 10.8 | -7.78 | 2.51 |
| $H_{+}$ | 19.9 | -8.20 | 1.55 |

(c) $\Delta$ of Value Function Differences

| $\left(\delta_{m}, Q\right)$ | $\Delta\left(\Delta_{2,3}^{1}\right)$ | $\Delta\left(\Delta_{m, 1}^{2}\right)$ | $\Delta\left(\Delta_{m, 1}^{2}\right)$ |
| :---: | :---: | :---: | :---: |
| $M_{0}-L_{0}$ | 8.58 | -1.42 | -0.78 |
| $H_{0}-L_{0}$ | 12.9 | -2.43 | -1.14 |
| $L_{+}-L_{0}$ | 12.0 | -14.0 | -9.30 |
| $M_{+}-M_{0}$ | 14.8 | -18.8 | -9.09 |
| $H_{+}-H_{0}$ | 19.5 | -18.2 | -9.75 |

Note: The quantity of money, $Q$, is $2 / 18(\mathrm{~L}), 6 / 18(\mathrm{M})$, and $10 / 18(\mathrm{H})$. The 0 and + subscripts of $L$, $M$, and $H$, mean $\delta_{m}=0$ and $\delta_{m}=0.08$, respectively.

Table 4: The 18-agent economy
(a) Value Functions

| $\left(\delta_{m}, Q\right)$ | $V_{1,2}$ | $V_{1,3}$ | $V_{1, m}$ | $V_{2,3}$ | $V_{2,1}$ | $V_{2, m}$ | $V_{3,1}$ | $V_{3,2}$ | $V_{3, m}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $L_{0}$ | 109 | 113 | N.A. | 116 | 141 | 153 | 195 | N.A. | 207 |
| $M_{0}$ | 83.3 | 84.3 | 146 | 86.4 | 113 | 124 | 157 | N.A. | 167 |
| $H_{0}$ | 41.6 | N.A. | 94.4 | 2.92 | 59.6 | 70.1 | 95.8 | N.A. | 106 |
| $L_{+}$ | 104 | 108 | 144 | 112 | 139 | N.A. | 191 | N.A. | 194 |
| $M_{+}$ | 63.4 | N.A. | 92.1 | 8.75 | 69.9 | 65.4 | 115 | N.A. | 118 |
| $H_{+}$ | 40.4 | N.A. | 50.4 | -5.88 | 46.3 | 41 | 79.5 | N.A. | 81.2 |

(b) Differences of Value Functions

| $\left(\delta_{m}, Q\right)$ | $\Delta_{2,3}^{1}$ | $\Delta_{m, 1}^{2}$ | $\Delta_{m, 1}^{3}$ |
| :---: | :---: | :---: | :---: |
| $L_{0}$ | -4.24 | 11.7 | 12.1 |
| $M_{0}$ | -2.91 | 10.3 | 11.3 |
| $H_{0}$ | N.A. | 10.3 | 11.0 |
| $L_{+}$ | -3.85 | N.A. | 3.69 |
| $M_{+}$ | N.A. | -5.46 | 2.51 |
| $H_{+}$ | N.A. | -4.99 | 1.89 |

(c) $\Delta$ of Value Function Differences

| $\left(\delta_{m}, Q\right)$ | $\Delta\left(\Delta_{2,3}^{1}\right)$ | $\Delta\left(\Delta_{m, 1}^{2}\right)$ | $\Delta\left(\Delta_{m, 1}^{3}\right)$ |
| :---: | :---: | :---: | :---: |
| $M_{0}-L_{0}$ | 1.33 | -1.42 | -0.84 |
| $H_{0}-L_{0}$ | N.A | -1.43 | -1.11 |
| $L_{+}-L_{0}$ | 0.39 | N.A | -8.41 |
| $M_{+}-M_{0}$ | N.A | -15.76 | -8.79 |
| $H_{+}-H_{0}$ | N.A | -15.29 | -9.11 |

Note: In panel (a) some values are not available (N.A.) because in the initial distribution of stocks nobody is in 9 that particular situation. For instance, in $L_{0}, V_{1, m}$ is N.A. because no type 1 agent carries fiat money (refer to table 6). See also the note of table 3 .

Table 5: Economy with Error Prone Agents ( $N=18$ )
(a) Value Functions

| $\left(\delta_{m}, Q\right)$ | $V_{1,2}$ | $V_{1,3}$ | $V_{1, m}$ | $V_{2,3}$ | $V_{2,1}$ | $V_{2, m}$ | $V_{3,1}$ | $V_{3,2}$ | $V_{3, m}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $L_{0}$ | 113 | 115 | N.A. | 112 | 139 | 148 | 192 | N.A. | 202 |
| $M_{0}$ | 83.8 | 84.7 | 141 | 80.6 | 109 | 118 | 151 | N.A. | 161 |
| $H_{0}$ | 42.7 | N.A. | 90.1 | 4.17 | 59.3 | 68.4 | 92.6 | N.A. | 103 |
| $L_{+}$ | 107 | 110 | 142 | 110 | 137 | N.A. | 189 | N.A. | 191 |
| $M_{+}$ | 64.6 | N.A. | 90.9 | 11.7 | 70.3 | 64.5 | 114 | N.A. | 117 |
| $H_{+}$ | 34.3 | N.A. | 54.2 | -4.72 | 45.3 | 39.8 | 77.0 | N.A. | 78.6 |

(b) Differences of Value Functions

| $\left(\delta_{m}, Q\right)$ | $\Delta_{2,3}^{1}$ | $\Delta_{m, 1}^{2}$ | $\Delta_{m, 1}^{3}$ |
| :---: | :---: | :---: | :---: |
| $L_{0}$ | -2.81 | 8.48 | 10.1 |
| $M_{0}$ | -1.10 | 8.22 | 9.66 |
| $H_{0}$ | N.A. | 8.99 | 10.1 |
| $L_{+}$ | -2.37 | N.A. | 2.51 |
| $M_{+}$ | N.A. | -6.37 | 2.24 |
| $H_{+}$ | N.A. | -5.83 | 1.62 |

(c) $\Delta$ of Value Function Differences

| $\left(\delta_{m}, Q\right)$ | $\Delta\left(\Delta_{2,3}^{1}\right)$ | $\Delta\left(\Delta_{m, 1}^{3}\right)$ | $\Delta\left(\Delta_{m, 1}^{2}\right)$ |
| :---: | :---: | :---: | :---: |
| $M_{0}-L_{0}$ | 1.71 | -0.26 | -0.54 |
| $H_{0}-L_{0}$ | N.A. | 0.51 | 0.0 |
| $L_{+}-L_{0}$ | 0.44 | N.A. | -7.59 |
| $M_{+}-M_{0}$ | N.A. | -14.6 | -7.42 |
| $H_{+}-H_{0}$ | N.A. | -14.8 | -8.48 |

Note: See notes of tables 3 and 4.

Table 6: Initial Distribution of Goods and Money

| Treatment | $n_{1,2}$ | $n_{1,3}$ | $n_{1, m}$ | $n_{2,3}$ | $n_{2,1}$ | $n_{2, m}$ | $n_{3,1}$ | $n_{3,2}$ | $n_{3, m}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $L_{0}$ | 4 | 2 | 0 | 3 | 2 | 1 | 5 | 0 | 1 |
| $M_{0}$ | 3 | 1 | 2 | 2 | 2 | 2 | 4 | 0 | 2 |
| $H_{0}$ | 3 | 0 | 3 | 1 | 2 | 3 | 2 | 0 | 4 |
| $M_{+}$ | 4 | 0 | 2 | 2 | 3 | 1 | 3 | 0 | 3 |
| $H_{+}$ | 3 | 0 | 3 | 1 | 2 | 3 | 2 | 0 | 4 |

Table 7: Frequencies of Offers in Laboratory Experiments

|  | Panel I Type 1 Offers |  |  | Panel II Type 2 Offers |  |  | Panel III. <br> Type 3 Offers |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Money for |  |  | Money for |  |  | Money for |  |  |
| Treatment | 1 | 2 | 3 | 2 | 3 | 1 | 3 | 1 | 2 |
| $L_{0}$ | 1.00 | 0.00 | 0.10 | 1.00 | 0.00 | 0.27 | 1.00 | 0.07 | 0.02 |
| $M_{0}$ | 0.97 | 0.06 | 0.03 | 0.96 | 0.00 | 0.14 | 0.98 | 0.13 | 0.08 |
| $H_{0}$ | 1.00 | 0.09 | 0.12 | 0.97 | 0.02 | 0.23 | 0.97 | 0.09 | 0.07 |
| $M_{+}$ | 1.00 | 0.05 | 0.16 | 0.97 | 0.02 | 0.27 | 0.98 | 0.31 | 0.25 |
| $H_{+}$ | 0.98 | 0.07 | 0.19 | 0.96 | 0.03 | 0.30 | 0.92 | 0.22 | 0.11 |
|  | Good 2 for |  |  | Good 3 for |  |  | Good 1 for |  |  |
| Treatment | 1 | 0 | 3 | 2 | 0 | 1 | 3 | 0 | 2 |
| $L_{0}$ | 0.99 | 0.77 | 0.40 | 0.98 | 0.79 | 0.98 | 0.98 | 0.73 | 0.12 |
| $M_{0}$ | 0.98 | 0.79 | 0.23 | 0.99 | 0.95 | 0.98 | 0.98 | 0.72 | 0.14 |
| $H_{0}$ | 1.00 | 0.84 | 0.24 | 1.00 | 0.90 | 1.00 | 1.00 | 0.89 | 0.15 |
| $M_{+}$ | 1.00 | 0.73 | 0.22 | 0.98 | 0.83 | 0.99 | 1.00 | 0.55 | 0.32 |
| $H_{+}$ | 1.00 | 0.69 | 0.14 | 1.00 | 0.92 | 1.00 | 0.97 | 0.48 | 0.27 |
|  | Good 3 for |  |  | Good 1 for |  |  | Good 2 for |  |  |
| Treatment | 1 | 0 | 2 | 2 | 0 | 3 | 3 | 0 | 1 |
| $L_{0}$ | 0.97 | 0.67 | 0.29 | 0.99 | 0.54 | 0.11 | 1.00 | 0.64 | 0.64 |
| $M_{0}$ | 1.00 | 0.75 | 0.69 | 1.00 | 0.62 | 0.02 | 1.00 | 0.53 | 0.48 |
| $H_{0}$ | 1.00 | 0.83 | 0.50* | 0.97 | 0.51 | 0.00 | 1.00* | 0.31 | 0.38 |
| $M_{+}$ | 1.00 | 0.50 | 0.44 | 0.98 | 0.42 | 0.04 | 1.00 | 0.38 | 0.58 |
| $H_{+}$ | 1.00 | 0.63 | 1.00* | 1.00 | 0.31 | 0.08 | 0.75* | 0.56 | 0.67 |

Notes: (*) fewer than 5 observations. Good 0 refers to fiat money

Table 8: Estimator and 95\% Confidence Interval (CI)

|  | Type 1 |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Treatment | $\hat{s}_{1}$ | CI | $\hat{s}_{2}$ | CI | $\hat{s}_{3}$ | CI |  |
| $L_{0}$ | 0.83 | $(0.76,0.87)$ | 0.79 | $(0.66,0.87)$ | 0.41 | $(0.35,0.46)$ |  |
| $M_{0}$ | 0.84 | $(0.81,0.87)$ | 0.84 | $(0.77,0.90)$ | 0.26 | $(0.20,0.34)$ |  |
| $H_{0}$ | 0.84 | $(0.80,0.87)$ | 0.76 | $(0.67,0.83)$ | 0.35 | $(0.24,0.46)$ |  |
| $M_{+}$ | 0.79 | $(0.75,0.83)$ | 0.77 | $(0.69,0.82)$ | 0.33 | $(0.27,0.40)$ |  |
| $H_{+}$ | 0.74 | $(0.70,0.78)$ | 0.79 | $(0.71,0.85)$ | 0.22 | $(0.13,0.33)$ |  |
| Type 2 |  |  |  |  |  |  |  |
| Treatment | $\hat{s}_{1}$ | CI | $\hat{s}_{2}$ | CI | $\hat{s}_{3}$ | CI |  |
| $L_{0}$ | 0.91 | $(0.85,0.95)$ | 0.66 | $(0.59,0.73)$ | 0.96 | $(0.94,0.98)$ |  |
| $M_{0}$ | 0.96 | $(0.93,0.98)$ | 0.72 | $(0.66,0.77)$ | 0.96 | $(0.93,0.98)$ |  |
| $H_{0}$ | 0.91 | $(0.88,0.94)$ | 0.59 | $(0.53,0.65)$ | 0.98 | $(0.92,0.99)$ |  |
| $M_{+}$ | 0.89 | $(0.85,0.92)$ | 0.49 | $(0.43,0.54)$ | 0.98 | $(0.95,0.99)$ |  |
| $H_{+}$ | 0.91 | $(0.88,0.94)$ | 0.42 | $(0.37,0.48)$ | 0.97 | $(0.91,0.99)$ |  |
|  |  |  |  |  |  |  |  |
| Treatment | $\hat{s}_{1}$ | CI | $\hat{s}_{2}$ | Type 3 | CI | $\hat{s}_{3}$ |  |
| $L_{0}$ | 0.84 | $(0.79,0.88)$ | 0.92 | $(0.85,0.96)$ | 0.18 | $(0.14,0.22)$ |  |
| $M_{0}$ | 0.79 | $(0.75,0.83)$ | 0.85 | $(0.79,0.90)$ | 0.19 | $(0.14,0.24)$ |  |
| $H_{0}$ | 0.85 | $(0.81,0.88)$ | 0.84 | $(0.78,0.89)$ | 0.23 | $(0.14,0.34)$ |  |
| $M_{+}$ | 0.61 | $(0.57,0.65)$ | 0.68 | $(0.62,0.74)$ | 0.34 | $(0.28,0.40)$ |  |
| $H_{+}$ | 0.59 | $(0.55,0.63)$ | 0.77 | $(0.69,0.83)$ | 0.26 | $(0.18,0.36)$ |  |

Table 9: Test of Hypotheses and $p$-values

| Hypothesis | $h_{0}$ | $z$ | $p$-value |
| :---: | :---: | :---: | :---: |
| H1 |  |  |  |
| (a) | $s_{3}\left(1, M_{0}\right) \geq s_{3}\left(1, L_{0}\right)$ | 2.99 | 0.001 |
| (b) | $s_{3}\left(1, H_{0}\right) \geq s_{3}\left(1, L_{0}\right)$ | 0.91 | 0.181 |
| (c) | $s_{3}\left(1, H_{0}\right) \geq s_{3}\left(1, M_{0}\right)$ | -1.27 | 0.898 |
| (d) | $s_{3}\left(1, H_{+}\right) \geq s_{3}\left(1, L_{+}\right)$ | 2.59 | 0.005 |
| H2 type 2 |  |  |  |
| (a) | $s_{2}\left(2, L_{0}\right) \leq s_{2}\left(2, M_{0}\right)$ | -1.21 | 0.887 |
| (b) | $s_{2}\left(2, L_{0}\right) \leq s_{2}\left(2, H_{0}\right)$ | 1.45 | 0.073 |
| (c) | $s_{2}\left(2, M_{0}\right) \leq s_{2}\left(2, H_{0}\right)$ | 3.1 | 0.001 |
| (d) | $s_{2}\left(2, M_{+}\right) \leq s_{2}\left(2, H_{+}\right)$ | 1.55 | 0.061 |
| H2 type 3 |  |  |  |
| (a) | $s_{1}\left(3, L_{0}\right) \leq s_{1}\left(3, M_{0}\right)$ | 1.65 | 0.050 |
| (b) | $s_{1}\left(3, L_{0}\right) \leq s_{1}\left(3, H_{0}\right)$ | -0.31 | 0.622 |
| (c) | $s_{1}\left(3, M_{0}\right) \leq s_{1}\left(3, H_{0}\right)$ | -2.22 | 0.987 |
| (d) | $s_{1}\left(3, M_{+}\right) \leq s_{1}\left(3, H_{+}\right)$ | 0.65 | 0.257 |
| H3 |  |  |  |
| (a) | $s_{3}\left(1, M_{0}\right) \leq s_{3}\left(1, M_{+}\right)$ | -1.41 | 0.921 |
| (b) | $s_{3}\left(1, H_{0}\right) \leq s_{3}\left(1, H_{+}\right)$ | 1.64 | 0.050 |
| H4 type 2 |  |  |  |
| (a) | $s_{2}\left(2, M_{0}\right) \leq s_{2}\left(2, M_{+}\right)$ | 5.82 | <0.001 |
| (b) | $s_{2}\left(2, H_{0}\right) \leq s_{2}\left(2, H_{+}\right)$ | 4.02 | <0.001 |
| H4 type 3 |  |  |  |
| (a) | $s_{1}\left(3, M_{0}\right) \leq s_{1}\left(3, M_{+}\right)$ | 6.13 | $<0.001$ |
| (b) | $s_{1}\left(3, H_{0}\right) \leq s_{1}\left(3, H_{+}\right)$ | 8.14 | <0.001 |

Table 10: Estimates with Two Binomial Distributions

| Treatment | Speculative Behavior |  |  | Money Acceptance |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | $\hat{q}_{1}$ | $\hat{p}_{1}$ | $\hat{s}_{3}(1)\left(\tilde{s}_{3}(1)\right)$ | $\hat{q}_{2}$ | $\hat{p}_{2}$ | $\hat{s}_{2}(2)\left(\tilde{s}_{2}(2)\right)$ | $\hat{q}_{3}$ | $\hat{p}_{3}$ | $\hat{s}_{1}(3)\left(\tilde{s}_{1}(3)\right)$ |
| $L_{0}$ | 0.27 | 0.17 | $0.35(0.41)$ | 0.63 | 0.09 | $0.60(0.66)$ | 0.80 | 0.07 | $0.76(0.84)$ |
| $M_{0}$ | 0.12 | 0.15 | $0.23(0.26)$ | 0.69 | 0.06 | $0.67(0.52)$ | 0.82 | 0.09 | $0.76(0.79)$ |
| $H_{0}$ | 0.34 | 0.12 | $0.38(0.35)$ | 0.67 | 0.09 | $0.64(0.59)$ | 0.90 | 0.08 | $0.83(0.85)$ |
| $M_{+}$ | 0.23 | 0.16 | $0.31(0.33)$ | 0.56 | 0.15 | $0.54(0.49)$ | 0.70 | 0.14 | $0.65(0.61)$ |
| $H_{+}$ | 0.05 | 0.12 | $0.16(0.22)$ | 0.60 | 0.13 | $0.58(0.42)$ | 0.69 | 0.15 | $0.64(0.59)$ |

Notes: $\hat{q}_{1}(T)$ is the probability a type 1 chooses good 3 over good $2 ; \hat{p}_{1}(T)$ is the probability that this individual deviates from such decision. $\hat{s}_{3}(T)$ is the average probability of speculative behavior; $\tilde{s}_{3}(T)$ is copied from table 8 , for comparison purposes.

Table 11: Normal Distribution and Comparison across Statistical Models

|  | $\bar{s}_{3}(1, T)$ | $\hat{s}_{3}(1, T)$ | $\tilde{s}_{3}(1, T)$ | $\bar{s}_{2}(2, T)$ | $\hat{s}_{2}(2, T)$ | $\tilde{s}_{2}(2, T)$ | $\bar{s}_{1}(3, T)$ | $\hat{s}_{1}(3, T)$ | $\tilde{s}_{1}(3, T)$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $L_{0}$ | 0.35 | 0.35 | 0.41 | 0.63 | 0.60 | 0.66 | 0.77 | 0.76 | 0.84 |
| $M_{0}$ | 0.28 | 0.23 | 0.26 | 0.72 | 0.67 | 0.52 | 0.77 | 0.76 | 0.79 |
| $H_{0}$ | 0.31 | 0.38 | 0.35 | 0.66 | 0.64 | 0.59 | 0.81 | 0.83 | 0.85 |
| $M_{+}$ | 0.31 | 0.31 | 0.33 | 0.53 | 0.54 | 0.49 | 0.62 | 0.65 | 0.61 |
| $H_{+}$ | 0.17 | 0.16 | 0.12 | 0.49 | 0.58 | 0.42 | 0.62 | 0.64 | 0.59 |
|  | $\tilde{v}_{1}=1.59$ |  |  | $\tilde{v}_{2}=2.79$ |  |  | $\tilde{v}_{3}=1.80$ |  |  |

[^6]
## Figures

Figure 1: Steady State Equilibria
(a) Continuum of Agents

(b) Finite Number of agents


Note. Abbreviations: Fundamental; Speculative; Full or Partial acceptance of money. In panel (b) equilibria are computed on the dots. In panel (a) they are computed on a much thinner grid. See table 1 for parameters.

Figure 2: Acceptability and the Quantity of Money, $Q$


Figure 3: Acceptability and the Inflation $\operatorname{Tax}(Q=1 / 3)$


Note: Two equilibria coexist when $\delta_{m} \in[0.041,0.065]$ (see also fig. 1a).

Figure 4: Welfare ( $Q=6 / 18$ )


Note: Two equilibria coexist when $\delta_{m} \in[0.041,0.065]$ (see also figure 1a).

Figure 5: A Sample of Laboratory Observations


Notes: Panel (a) shows that in treatment $H_{0}, 19$ type 1 players always played fundamental strategies, and 8 always played speculative strategies. Panel (b) shows that in treatment $M_{+}, 15$ type 2 agents always used money to buy good 1 , and 13 never did. A few individuals did not consistently play one strategy or the other.

## Online Appendix A

## A. 1 Dynamics in the Baseline Model

This section reports the equations that describe the evolution of commodities and fiat money in the baseline economy.

To simplify notation we set $p_{i, j}^{\prime}=p_{i, j}(t+1)$ and drop the time argument for $p_{i, j}$ and $s_{j, k}^{i}$. We also denote with $q_{i, j}$ the stock of commodities after government confiscation and before matching and trading. We obtain the evolution equations

$$
\begin{gather*}
q_{i, i+1}=p_{i, i+1}+\delta_{m} p_{i, m}-\delta_{g} p_{i, i+1} \\
q_{i, i+2}=p_{i, i+2}-\delta_{g} p_{i, i+2}  \tag{12}\\
q_{i, m}=p_{i, i+1}-\delta_{m} p_{i, m}+\delta_{g}\left(p_{i, i+1}+p_{i, i+2}\right) \\
p_{i, i+1}^{\prime}=q_{i, i+1}+\left\{\sum_{i^{\prime}} \sum_{k} q_{i, k} q_{i^{\prime}, i+1} s_{k, i+1}^{i} s_{i+1, k}^{i^{\prime}}+\sum_{i^{\prime}} q_{i, k} q_{i^{\prime}, i} s_{i, k}^{i^{\prime}}-\sum_{i^{\prime}} \sum_{k} q_{i, i+1} q_{i^{\prime}, k} s_{i+1, k}^{i} s_{k, i+1}^{i^{\prime}}\right\} \\
p_{i, i+2}^{\prime}=q_{i, i+2}+\left\{\sum_{i^{\prime}} \sum_{k} q_{i, k} q_{i^{\prime}, i+2} s_{k, i+2}^{i} s_{i+2, k}^{i^{\prime}}-\sum_{i^{\prime}} \sum_{k} q_{i, i+2} q_{i^{\prime}, k} s_{i+2, k}^{i} s_{k, i+2}^{i^{\prime}}\right\} \\
p_{i, m}^{\prime}=q_{i, i+1}+\left\{\sum_{i^{\prime}} \sum_{k} q_{i, k} q_{i^{\prime}, m} s_{k, m}^{i} s_{m, k}^{i^{\prime}}-\sum_{i^{\prime}} \sum_{k} q_{i, m} q_{i^{\prime}, k} s_{m, k}^{i} s_{k, m}^{i^{\prime}}\right\} . \tag{13}
\end{gather*}
$$

The top equation of (12) ( $q_{i, i+1}$ ) represents the amount of fiat money that the government confiscates from money holders $\left(p_{i, m}\right)$ and the amount of goods $i+1$ it purchases from type $i$ agents. We assume that the government operates with a balanced budget, therefore we have $\delta_{g}=\frac{\delta_{m} Q}{1-Q}$.

Regarding the top equation of (13) ( $p^{\prime} i, i+1$ ), the first two sums inside the brackets on the left-hand side correspond to events that increase the share of individuals $i$ holding good $i+1$. Specifically, the term $q i, k q_{i^{\prime}, i+1}$ represents the probability that a type $i$ individual with good $k$ encounters a type $i^{\prime}$ individual with good $i+1$, and $s_{k, i+1}^{i} s_{i+1, k}^{i^{\prime}}$ calculates the probability that they will agree to exchange holdings. The term $q_{i, k} q_{i^{\prime}, i}$ takes into account the residual scenario in which a type $i$ individual with good $k$ meets a type $i^{\prime}$ individual with good $i$. In this case, trade occurs with probability $s_{i, k}^{i^{\prime}}$ because type $i$ always accepts good $i$. The third sum accounts for events that decrease $p_{i, i+1}^{\prime}$.

By setting $p_{i, j}^{\prime}=p_{i, j}$ in (13) and (12), we obtain a set of equations of stock distribution in the steady state. According to Bonetto and Iacopetta [2019], these equations always have at least one solution.

We turn now to the evolution of the value functions $V_{i, j}(t)$. The time index has been dropped and and $\left.V_{i, j}^{\prime}=V_{i, j}(t+1)\right)$. We have that

$$
\begin{equation*}
\tilde{V}_{i, j}=\sum_{i^{\prime}, k} p_{i^{\prime}, j} \sigma_{k, j}^{i} s_{k, j}^{i^{\prime}}\left(\left(1-\Delta_{k, i}\right) V_{i, k}+\Delta_{k, i}\left(V_{i, i+1}+u\right)\right)+\sum_{i^{\prime}, k} p_{i^{\prime}, j}\left(1-\sigma_{k, j}^{i} s_{k, j}^{i^{\prime}}\right) V_{i, j}-c_{j} \tag{14}
\end{equation*}
$$

and

$$
\begin{align*}
V_{i, i+1}^{\prime} & =(1-\rho)\left(\delta_{g} \widetilde{V}_{i, m}+\left(1-\delta_{g}\right) \tilde{V}_{i, i+1}\right) \\
V_{i, i+2}^{\prime} & =(1-\rho)\left(\delta_{g} \widetilde{V}_{i, m}+\left(1-\delta_{g}\right) \widetilde{V}_{i, i+2}\right)  \tag{15}\\
V_{i, m}^{\prime} & =(1-\rho)\left(\delta_{m}\left(\widetilde{V}_{i, i+1}-D\right)+\left(1-\delta_{g}\right) \widetilde{V}_{i, m}\right)
\end{align*}
$$

where $\Delta_{i, j}$ is the Kronecker symbol, that is $\Delta_{i, i}=1$ while $\Delta_{i, j}=0$ if $i \neq j$.
Equation (14) accounts for the variation in the value functions resulting from matching and trade. The first sum on the right-hand side represents the expected flow of utility for an agent of type $i$ with good $j$, who plays strategy $\boldsymbol{\sigma}^{i}$, given they meet an agent $i^{\prime}$ who has good $k$. This meeting occurs with probability $p_{i^{\prime}, k}$ and trade occurs with probability $\sigma_{k, j}^{i} s_{j, k}^{i^{\prime}}=1$. If trade occurs, agent $i$ leaves the meeting with good $k$ and its continuation value $V_{i, k}$, or with good $i+1$ and consumption utility $u$, if $k=i$. If no trade occurs, agent $i$ remains with good $j$. Finally, $c_{j}$ is the cost of storing good $j$.

Equation (15) accounts for the variation in the value functions resulting from the government's seigniorage policy. For instance, the term $\delta_{g} \tilde{V}_{i, m}$ in the first equation represents the expected continuation value of an agent of type $i$ who meets government agents and sells their good $j$ for fiat money.

## A. 2 The KW model with Finite Number of Agents

The economy consists of $N \in \mathbb{N}$ agents, each indicated by $a \in 1, \ldots, N$. The agents are divided into three groups of equal size, with the first, second, and third groups being of type 1,2 , and 3 , respectively. The type of agent $a$ is indicated by $i(a) \in 1,2,3$. The state of the economy at time $t$ is represented by $X=\left(x_{1}, x_{2}, \ldots, x_{N}\right) \in 1,2,3, m^{N}$, where $x_{a}$ represents the type of good held by agent $a$, and each individual holds only one good at a time. The set of possible holdings, denoted by $\Omega_{Q} \subset 1,2,3, m^{N}$, consists of all $X$ such that $x_{a} \neq i(a)$ and $Q N$ agents have $x_{a}=m$. The number of agents of type $i$ holding good $j$ is represented by $n_{i, j}$, and $p_{i, j}=n_{i, j} / N$.

As described in Section 2, in each period, an agent holding fiat money must give it away to a government agent with probability $\delta_{m}$. Then, all agents are randomly paired for possible trades. Note that, since the agents are equally divided among the three groups and every agent participates in a trade in each period, $N$ must be a multiple of 6 . The strategy of agent $a$ when trading good $j$ for good $k$ is represented by $s_{j, k}^{a}$. If $s_{j, k}^{a}=1$, agent $a$ is favorable to the trade, and if it is 0 , agent $a$ is not. The complete characterization of agent $a$ 's strategy is given by the vector $\mathbf{s}^{a}=\left(s_{i+1, m}^{a}, s_{i+2, m}^{a}, s_{i+1, i+2}^{a}\right)$, where $i=i(a)$ and $\mathbf{s}^{a} \in \Sigma$. Note that for simplicity, we only consider the case where the strategy $\mathbf{s}^{a}$ of agent $a$ is fixed, as opposed to depending on the state $X$ of the economy. The distinction between fixed $\mathbf{s}^{a}$ and state-dependent $\mathbf{s}^{a}$ is similar to the distinction between open-loop and closed-loop Nash equilibria discussed in Fudenberg and Levine (1988).

For the baseline economy, the stock holdings $\mathbf{p}(t)$ at time $t$ can be calculated to determine the holdings $\mathbf{p}(t+1)$ at time $t+1$ through equations (12) and (13). The random nature of the matching process in a small economy, however, makes this calculation impossible, requiring us to adopt a probabilistic approach.

The probability measure $\mathbf{P}_{0}$ describes the initial state of the economy. Specifically,
it gives the probability of finding the system in state $X$ at time $t=0$ through $\mathbf{P}_{0}(X)$. Given a set of strategies $\hat{\mathbf{s}}=\left(\mathbf{s}^{1}, \ldots, \mathbf{s}^{N}\right) \in \Sigma^{N}$, the matching and sequestration processes uniquely determine the sequence of evolved probability measures $\mathbf{P}_{t}$ for all future times $t>0$. The probability of finding the economy in state $X$ at time $t$ is represented as $\mathbf{P}_{t}(X)$. We assume that there exists a unique invariant probability measure, denoted $\overline{\mathbf{P}}$.

Let $V_{a}(X, \hat{\mathbf{s}})$ be the expected discounted value of agent $a$ when the economy is in a state $X$. This function is defined in appendix A.2. A set of strategies $\hat{\mathbf{s}} \in \Sigma^{N}$ is referred to as a Nash Equilibrium if equation (16) holds for every agent $a$ :

$$
\begin{equation*}
\mathbb{E} 0\left(V_{a}\left(X, \mathbf{s}^{a}, \hat{\mathbf{s}}-a\right)\right)=\max _{\boldsymbol{\sigma}^{a} \in \Sigma} \mathbb{E} 0\left(V_{a}\left(X, \boldsymbol{\sigma}^{a}, \hat{\mathbf{s}}-a\right)\right) \tag{16}
\end{equation*}
$$

In this equation, $\hat{\mathbf{s}}_{-a}$ represents the set of strategies of all agents excluding agent $a$, and $\mathbb{E}_{0}$ is the average with respect to the agents' knowledge of the initial state of the economy, represented by the probability measure $\mathbf{P}_{0}$ on $\Omega_{Q}$. As initial $\mathbf{P}_{0}(X)$, we consider both the invariant distribution $\overline{\mathbf{P}}$ and a unit mass $\mathbf{P}_{X_{0}}$ on the state $X_{0} \in \Omega_{Q}$. This coincides with the starting distribution of the lab experiments (see table 6 and section 3.1). Also, since $\mathbf{s}^{a}$ only depends on the type $i(a)$ of agent $a$, we can denote $\mathbf{s}^{i}$ as the strategies and $V_{i}(X, \mathbf{s})$ as the value functions of any agent of type $i$. We can now define a transition kernel $\mathcal{K}: \Omega_{M} \times \Omega_{M} \rightarrow[0,1]$ where $\mathcal{K}\left(X, X^{\prime}\right)$ is the probability for the system to go from $X$ to $X^{\prime}$ after the matching, consumption and sequestration is completed. Clearly $\mathcal{K}$ depends on the strategies $\mathbf{s}^{a}$ of all the agents. It is difficult to give an explicit expression of $\mathcal{K}$; yet we do not need it in the coming discussion.

Since the evolution is intrinsically stochastic, it is natural to study the evolution of the probability measure $\mathbf{P}_{t}$, where $\mathbf{P}_{t}(X)$ is the probability of finding the system in the state $X \in \Omega_{M}$ at time $t$. Given the transition kernel $\mathcal{K}$, this evolution can be written has

$$
\begin{equation*}
\mathbf{P}_{t+1}(X)=\sum_{X^{\prime} \in \Omega} \mathcal{K}\left(X, X^{\prime}\right) \mathbf{P}_{t}\left(X^{\prime}\right) \tag{17}
\end{equation*}
$$

This describes a Markov chain on $\Omega_{M}$. Since $\Omega_{M}$ is a finite set, it is easy to see that this evolution always admit at least one steady state probability measure $\overline{\mathbf{P}}$ satisfying $\sum_{X^{\prime} \in \Omega} \mathcal{K}\left(X, X^{\prime}\right) \overline{\mathbf{P}}\left(X^{\prime}\right)=\overline{\mathbf{P}}(X)$. We assume that $\overline{\mathbf{P}}$ is unique. Uniqueness, together with ergodicity and exponential mixing, are well verified by our numerical simulations.

Let $n_{i, j}(t)$ be the number of agents of type $i$ carrying good $j$ at time $t$, and let $p_{i, j}(t)=n_{i, j}(t) / N .{ }^{8}$ Given the strategies $\hat{\mathbf{s}} \in \Sigma^{N}$ played by the agents, we define the integrated discounted utility functions $V_{a}(X, \hat{\mathbf{s}})$ for agent $a$ in the state $X$ as:

$$
\begin{equation*}
V_{a}(X, \hat{\mathbf{s}})=\sum_{t \geq 0}(1-\rho)^{t} \sum_{X^{\prime}} v_{a}\left(X^{\prime}\right) \mathbf{P}_{t}\left(X^{\prime}\right) \tag{18}
\end{equation*}
$$

where $\mathbf{P}_{t}$ is the probability measure at time $t$ with $\mathbf{P}_{0}=\mathbf{P}_{X}$, the unit mass on $X$, and $v_{a}(X)$ is the average flow of utility to agent $a$ in the state $X$. This is given by their storage

[^7]$\operatorname{cost} c_{x_{a}}$ plus the probability of obtaining their consumption good:
$$
v_{a}(X)=-c_{x_{a}}+\frac{U-D}{N-1} \sum_{a^{\prime}} \Delta_{x_{a^{\prime}, i(a)}} s_{i(a), x_{a}}^{a^{\prime}}-\delta_{g} \Delta_{x_{a}, m} D .
$$

It is important to note that $V_{a}(X, \hat{\mathbf{s}})$ will generally depend on the strategies played by all agents.

Obtaining a comparable equation for the value function's evolution equation presents a challenge. Also a description of the best response becomes increasingly intricate as the behavior of a single agent affects the overall economy.

## A. 3 More on Robustness of Statistical Analysis

This section outlines the process for obtaining the estimates presented in table 10 and offers further interpretation of its data.

Equation (7) indicates that, for individual a of type 1 in T , the following holds:

$$
\begin{array}{r}
\mathbb{E}\left(\left.\frac{n_{1}(a, T)}{N_{1}(a, t)}-m_{1}(T) \right\rvert\, N(a, T)>0\right)=0  \tag{19}\\
\mathbb{E}\left(\left.\frac{n_{1}(a, T)\left(N_{1}(a, T)-n_{1}(a, T)\right)}{N(a, T)(N(a, T)-1)}-v_{1}(T) \right\rvert\, N(a, T)>1\right)=0 .
\end{array}
$$

where

$$
\begin{align*}
m_{1}(T) & =q_{1}(T)\left(1-p_{1}(T)\right)+\left(1-q_{1}(T)\right) p_{1}(T) \\
v_{1}(t) & =q_{1}(T)\left(1-q_{1}(T)\right) . \tag{20}
\end{align*}
$$

We use the Generalized Method of Moments to estimate $m_{1}(T)$ and $v_{1}(T)$ by substituting the expectations in equation (19) with the average frequency of type 1 agents accepting good 3 in exchange for good 2. By inverting equation (20), we calculate estimators for $p_{1}(T)$ and $q_{1}(T)$ as $\hat{p}_{1}(T)$ and $\hat{q}_{1}(T)$. With these estimators, we derive $\hat{s}_{3}(1, T)$, which is equal to $\hat{p}_{1}(T) \hat{q}_{1}(T)+\left(1-\hat{p}_{1}(T)\right)\left(1-\hat{q}_{1}(T)\right)$. We use probability distributions of the type in equation (9) to obtain further estimators for $\hat{q}_{2}(T), \hat{q}_{3}(T), \hat{p}_{2}(T)$, and $\hat{p}_{3}(T)$.

A Test of Nash-like Behavior. The experiments showed that some type 2 individuals exhibit behavior that aligns with the predictions of the model, which we refer to as Nashlike behavior. This doesn't necessarily imply that these individuals are following complex calculations, but rather may suggest they have some sense of how changes in their payoffs are tied to the distribution of goods and fiat money. For example, table 10 shows that $69 \%$ of type 2 players chose money over good 1 in $M_{0}$, and $56 \%$ in $M_{+}$. The difference between these two values indicates that at least $13 \%$ of type 2 players exhibit Nash-like behavior by consistently choosing the strategy predicted by the model for an infinite number of agents in that particular parameterization. The fraction of individuals who exhibit Nashlike behavior can be larger than $13 \%$. This is because for each type 2 individual who decreases their acceptance of money from $M_{0}$ to $M_{+}$, there may be one who does not accept money in $M_{0}$, but does in $M_{+}$, a pattern we refer to as Nash-orthogonal behavior. As a result, the $13 \%$ must be viewed as net of the actions of the Nash-orthogonal agents.

Since these individuals can be at most $31 \%$ of the total - those who do not accept money for good 1 in $M_{0}$, but do accept it in $M_{+}$, - no more than $44 \%$ of type 2 players display Nash-like behavior. To get a more accurate estimate of the consistency of Nash-like behavior, we can use the trading decisions of agents who played type 2 in both $M_{0}$ and $M_{+}$. Proceeding as for eqs. (9) and (19) we obtain

$$
\left.\left.\mathbb{E}\left(\frac{n_{2}\left(a, M_{0}\right) n_{2}\left(a, M_{+}\right)}{N_{2}\left(a, M_{+}\right) N_{2}\left(a, M_{+}\right)}-\rho\left(M_{0}, M_{+}\right)\right) \right\rvert\, N\left(a, M_{0}\right)>0 \& N\left(a, M_{+}\right)>0\right)=0
$$

where

$$
\begin{align*}
\rho\left(M_{0}, M_{+}\right)= & \left(q_{2}\left(M_{0}\right) q_{2}\left(M_{+}\right)+c\right) p_{2}\left(M_{0}\right) p_{2}\left(M_{+}\right)+ \\
& \left(q_{2}\left(M_{0}\right)\left(1-q_{2}\left(M_{+}\right)\right)-c\right) p_{2}\left(M_{0}\right)\left(1-p_{2}\left(M_{+}\right)\right)+  \tag{21}\\
& \left(\left(1-q_{2}\left(M_{0}\right)\right) q_{2}\left(M_{+}\right)-c\right)\left(1-p_{2}\left(M_{0}\right)\right) p_{2}\left(M_{+}\right)+ \\
& \left(\left(1-q_{2}\left(M_{0}\right)\right)\left(1-q_{2}\left(M_{+}\right)\right)+c\right)\left(1-p_{2}\left(M_{0}\right)\right)\left(1-p_{2}\left(M_{+}\right)\right)
\end{align*}
$$

with $c=c\left(M_{0}, M_{+}\right)$. The expression $\left(q_{2}\left(M_{0}\right) q_{2}\left(M_{+}\right)+c\right)$ in (21) represents the fraction of agents that accept money in both $M_{0}$ and $M_{+}$, while $\left(q_{2}\left(M_{0}\right)\left(1-q_{2}\left(M_{+}\right)\right)-c\right)$ represents the fraction of agents that accept money in $M_{0}$ but not in $M_{+}$, referred to as Nash-like agents. Equations (20) and (21) indicate that the joint distribution of type 2's money acceptance over good 1 in treatments $M_{0}$ and $M_{+}$can be fully described with the extra parameter $c\left(M_{0}, M_{+}\right)$. Inverting (21) and using our data yields an estimate of $\hat{c}\left(M_{0}, M_{+}\right)=0.15$, indicating that approximately $15 \%$ of type 2 agents favor fiat money over good 1 in $M_{0}$ but not in $M_{+}$. Conversely, only $2 \%$ of type 2 agents exhibit Nashorthogonal behavior. The remaining $83 \%$ of players maintain the same strategies in both $M_{0}$ and $M_{+} .{ }^{9}$ To summarize, type 2 's responses to inflation, inferred from the their choices in $M_{0}$ and $M_{+}$, is highly correlated: most agents ( $83 \%$ ) stick to the same strategy irrespective of the inflation level; A significant but small number of them (15\%) follows a Nash-like behavior; Only a very small fraction (2\%) appear to adopt a Nash-orthogonal behavior. We reached similar conclusions when type 2 take decisions in $H_{0}$ and $H_{+}$and when we studied the behavior of type 3 agents.

[^8]
## Online Appendix B

## B. 1 Rules and Objectives of the Game

This is an experiment about decisions related to transactions. Please read the explanation below carefully. If you have any questions, please quietly raise your hand and the experimenter will assist you.

In this experiment, you will play the same game several times, with each iteration consisting of multiple periods. Three types (roles) of players will decide whether to trade goods or not.

At the beginning of each game, participants will be randomly assigned a role (A, B, or C), with an equal number of players assigned to each role. Your assigned role will be displayed on the computer screen and will remain unchanged throughout the game. We will refer to participants who are given the role of $A, B$, and $C$, as Type $A, B$, and $C$ players, respectively.

The game involves three types of goods (good 1, good 2, and good 3) and tokens. Each type of player can earn 130 points by obtaining the desired good through exchange and consuming it.

- Type A Players desire good 1.
- Type B Players desire good 2.
- Type C Players desire good 3.

Obtaining a token does not result in earning points, but it can be used in further exchange.

At the beginning of the game, every player receives 150 points, one unit of good of a type different from the desired good, or a token. For example, a Type A Player receives one unit good 2, or of good 3, or one token. A Type B Player receives one unit of good 1 , or of good 3, or one token. A Type C Player receives one unit of good 1, or of good 2, or one token.

Once the game starts, participants will be randomly paired into pairs and informed on the computer screen of their current good, the type of the player they are matched with and the type of good of their partner has.

Based on this information, you can decide whether you want to trade goods with your partner. Transactions are always based on one unit and if both players agree to trade, the transaction takes place.

As the result of the transaction, if you obtain the good you desire, you will consume it and obtain 130 points. At the same time, you will have to pay 30 points to produce one unit of a good. Consumption and production occur automatically in the experiment resulting in a net gain of 100 points for obtaining the desired good.

The type of a good players can produce depend on their type. Namely,

- Type A Player can produce good 2,
- Type B Player can produce good 3, and
- Type C Player can produce good 1.

The cost of producing a good is a constant 30 points across all types.
Once participants have made their transactions, and resulting exchanges, consumption, and production are completed, the computer randomly determines whether to confiscate tokens. Specifically, a player who holds a token, looses this with probabuility p, because of confiscation. If your toke is confiscated, you will have to pay 30 points to produce the good you are able to produce. The probability of confiscation p \% remains constant throughout the game and is shown at the top of the screen.

At the same time, if someone's token is confiscated, then the same number of players as the number of confiscated tokens will be selected randomly among those players who hold goods other than token. Holding of each of these selected players will be automatically replaced by a token. Information about confiscation and replacement will also be displayed on your screen.

Once this process is done, a period of a game ends. At the end of a period, the game ends with $10 \%$ probability. The game continues to the next period with $90 \%$ probability. The total points you have at the end of the game will be the number of points you have earned for that game.

If the game continues to the next period, you will have to pay a storage cost. The storage cost depends on the type of good you hold as follows:

- Token: 0 points
- Good 1: 4 points
- Good 2: 10 points
- Good 3: 20 points.

These storage cost will be automatically deducted from your accumulated points when the next period starts. Thus, when you are deciding whether to trade or not, in addition to whether you can earn the point by consuming your opponent's holding, you may want to consider the storage cost you may have to pay in case the game continues to the next period.

The following figure summarize the flow within in a period of a game.

Figure 6: Flow diagram


While this is the way the game proceeds, during the experiment, it will be implemented in the following manner.

A game will be played in blocks of 10 periods. Therefore, even if the game has ended during the first period, you will not be informed when the game has ended until the end of the block of 10 periods. At the end of the block, whether the game has ended during the block or not will be displayed on your screen. If the game has not ended, it will continue to the next 10-period block. If the game has ended during the one of the rounds in past 10 -period block, the screen will show when the game has ended. We will also show you the list of random numbers that have been generated at the end of each period to determine the termination of the game. Each of these random numbers is a number between 0 and 1 , and when the number is less than 0.1 , the game ends.

The point you earn for the game is based on the point you had at the end of the period when the game has ended. For example, even if you played the game for two 10-period blocks (thus 20 periods in total), if the game ended in 12th period, your point for this game is the point you had at the end of 12th period.

Once the whole experiment end, one of the games will be chosen randomly for payment. In addition to the participation compensation of 500 JPY , you will be paid based on the number of points you have earned in the chosen game with an exchange rate of 1 point $=$ 10 JPY. If the points you have earned in the chosen game is negative, then you will only receive the participation compensation of 500 JPY .

## B. 2 Screenshots

This document explains the content of the screens the participants see during the experiment.

This is the first and main screen.
Figure 7: Main screen


This screen:
(1) Shows your type, current period, and the probability with which token is confiscated during this game.
(2) The points you earn by acquiring a commodity, and the storage costs for good 1,2 , and 3. The storage cost of a token is zero.
(3) The points you have earned so far in the current game, the type of good you currently hold, the type of the player with whom you are matched in the current period, and the type of good this player carries.
(4) Shows a "Yes" and a "No" button you have to use for your trading decision. If you would like to exchange your holding with that of your partner, press "Yes", otherwise press "No".
(5) Shows the probability the game ends in the current period or in future periods. Recall that there is a $10 \%$ probability the game is over at the end of the current period.
(6) Shows the type of goods each type of players hold. The tag "Current" shows the distribution of goods across players in the current period. The tag "History" shows the average distribution of goods across players counting from beginning of the game.

Please decide whether you want to trade or not and press either "Yes" or "No" button.

Once all players have communicated their choices the following screen appears:
Figure 8: Screen 2


The above screen has two main sections that show
(7) Whether you exchanged your holding with your partner,
(8) The good you are currently holding, and the points you accumulated so far.

After reviewing the above information, please press OK to continue.

Once everyone has pressed OK the following screen appears：
Figure 9：Screen 3


合計で単位のトークンカ没収されました。（9）


The above screen shows
（9）How many tokens have been confiscated in this period（across all 18 players），and
（10）Your current holding；this will differ from that in the previous screen only if your token has been confiscated．

Once everyone presses OK, the following screen appears:
Figure 10: Screen 4


The above screen indicates whether
(11) The game is over in the current 10-period, or continues to the next 10-period block.
(12) The random termination numbers the computer generated from the beginning of the 10-period block. The game ends in the period the random number is smaller then 0.1.

## B. 3 Comprehension Quiz

1. Imagine you are a Type A Player in the game. By obtaining which type of good, would you gain 130 points?
Token, Good 1, Good 2, Good $3 \quad$ (correct answer good 1)
2. Imagine you are a Type B Player in the game. Which type of good can you produce? Token, Good 1, Good 2, Good $3 \quad$ (correct answer good 3)
3. How many points does it cost to produce any of the goods?
—— points (correct answer 30 points)
4. Imagine you have good 2 at the end of the current period. If the game continues to the next period, how many points do you have to pay for the storage cost?
—— points (correct answer 10 points)
5. Imagine you have a token at the end of the current period. If the game continues to the next period, how many points do you have to pay for the storage cost?
—— points (correct answer 0 points)
6. Imagine you obtained a token in an a trade in the current period. Is it possible that you will hold a good instead of the token at the start of next period?

- Not possible.
- Possible.
- Depending on the value of confiscation probability p.
(correct answer, depending on the value of confiscation probability p : if $\mathrm{p}=0$, then it is not possible, if $\mathrm{p}>0-$ it is possible.)

7. You have played a game for 20 periods. At the end of the 20 th period, you have 240 points. The next screen shows that the game ended at the 14th period, when you had accumulated 200 points. How many points do you earn in this game?

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[^1]:    ${ }^{1}$ Diercks (2019) lists 161 optimal monetary papers published since the mid-1990s that study the costs and benefits of inflation. Of these about half propose an optimal rule with a positive rate of inflation, 31 embrace some form of Friedman's rule, which requires negative inflation, and 50 are for an inflation close to zero. John Law's Money and Trade is widely regarded as the first modern examination of the crucial role of money, particularly banknotes, in driving trade and production. For a discussion, see, for example, Goetzmann, Chapter 20.

[^2]:    ${ }^{3}$ An agent of type $i$ has eight possible trading choices. However, the use of a simple transitivity trading rule, such as, if $\sigma_{2,3}^{1}=0$ and $\sigma_{2, m}^{1}=1$, then $\sigma_{3, m}^{1}=1$, narrows down their choices to the six options within $\Sigma$.

[^3]:    ${ }^{4}$ The characterization of equilibria does not consider type 3's acceptability of money for simplicity. The source code is available at https://bonetto.math.gatech.edu/KW

[^4]:    ${ }^{5}$ According to Lucas (2000), who used a version of the Sidrauski model, the average household's welfare cost of inflation is modest. The cost is measured by the missed gain of holding an interest-bearing asset, as there are no benefits derived from money's role as a means of payment. As he puts it, "it is in everyone's private interest to try to get someone else to hold non-interest-bearing cash and reserves" (Lucas, 2000, p. 247). Lucas's estimates indicate that a one percent increase in the nominal interest rate, attributed to inflation, would result in a welfare reduction of around 0.012 percent of GDP for representative households on the steady-state equilibrium. Later studies, including Kurlat (2019), argued inflation costs could reach almost 0.1 percent of GDP when considering the banks' operating costs.
    ${ }^{6}$ We obtained the numerical results using Octave and C codes. A graphical interface and the sources of the codes are available at https://bonetto.math.gatech.edu/KW and https://bonetto.math.gatech.edu/KW/Sources.

[^5]:    ${ }^{7}$ The statistical model of section 5 can be obtained as special case of the one outlined here by setting $b_{i}(a)=0$. With this restriction, we have, for example, $s_{3}(1, T)=\Phi\left(D_{1}(T)\right)$.

[^6]:    Notes: The columns $\bar{s}_{i}, \hat{s}_{i}$, and $\tilde{s}_{i}$, report probit estimates obtained with the normal distribution model, the two-binomial distribution model (table 10), and the baseline one (table 8), respectively. The $\tilde{v}_{i}$ is the estimated variance of $b_{i}(a)$ for the probit model.

[^7]:    ${ }^{8}$ We observe that the $p_{i, j}(t)$ are random variables with non-zero variance. In fact, we expect the variance of $p_{i, j}(t)$ to be of the order of $1 / \sqrt{N}$. This means that for small systems, such as those studied in the lab, the economy can spend significant amounts of time far from the steady state of the infinite system.

[^8]:    ${ }^{9}$ As a robustness exercise we replicated the estimates, under the assumption that type 2's decisions about money acceptance are independent in treatments $M_{0}$ and $M_{+}$. The fraction of the Nash-like agents is $q_{2}\left(M_{0}\right)\left(1-q_{2}\left(M_{+}\right)\right)=0.30$ and that of the Nash-orthogonal agents $q_{2}\left(M_{0}\right)\left(1-q_{2}\left(M_{+}\right)\right)=0.17$. Thus, there is little support for the independence assumption.

