No books or notes allowed. No laptop, graphic calculator or wireless devices allowed. Write clearly.

Name: ____________________

Question:	1	2	3	4	5	6	7	8	Total
Points:	5	7	17	10	17	10	10	24	100
Score:									

1. (5 points) Find the domain of the following function:

$$f(x) = \frac{1}{\sqrt{2x+1}}$$

Solution: We need $x \geq -1/2$ for $\sqrt{2x+1}$ to exist. But $x \neq -1/2$ if not we havce a division by 0. So that we have:

$$D(f) = \left(-\frac{1}{2}, \infty\right) \tag{1}$$

2. (7 points) Let $f(x)$ be defined by:

$$f(x) = \begin{cases} x^2 - 1 & \text{for } x < 3 \\ 2ax & \text{for } x \geq 3 \end{cases}$$

Find the values of a for which f is continuous.

Solution: The function is clearly continuous for every $x \neq 3$. At $x = 3$ we have

$$\lim_{x\to 3^-} f(x) = \lim_{x\to 3^-}(x^2 - 1) = 8 \tag{2}$$

$$\lim_{x\to 3^+} f(x) = \lim_{x\to 3^+}(2ax) = 6a \tag{3}$$

so that we need $6a = 8$ or

$$a = \frac{4}{3} \tag{4}$$

3. Compute the indicated limits.

   (a) (7 points)

$$\lim_{x\to 3} \frac{x^2+x-12}{x-3}$$

Solution: Since $x^2+x-12=(x-3)(x+4)$ we have

$$\lim_{x\to 3} \frac{x^2+x-12}{x-3} = \lim_{x\to 3} \frac{(x-3)(x+4)}{x-3} = \lim_{x\to 3} x+4 = 7 \tag{5}$$

   (b) (10 points)

$$\lim_{x\to 0} \frac{\cos^2(x)-1}{x^2}$$

Solution: Since $\cos^2(x)-1=-\sin^2(x)$ we have

$$\lim_{x\to 0} \frac{\cos^2(x)-1}{x^2} = -\lim_{x\to 0} \frac{\sin^2(x)}{x^2} = -\left(\lim_{x\to 0} \frac{\sin(x)}{x}\right)^2 = -1 \tag{6}$$

4. (10 points) Is there a root of the equation

$$2x^3+x^2+2=0$$

in the interval $[-2,-1]$?

Solution: Let $f(x)=2x^3+x^2+2$. We have

$$f(-2)=-10 \qquad f(-1)=1 \tag{7}$$

Since $-10<0<1$ from the Intermediate Value Theorem we know that there is a c in $[-2,-1]$ such that $f(c)=0$. c is a root of the equation.

5. Compute the following derivatives.

(a) (7 points)

$$f(x) = \tan(x)$$

Solution:

$$f'(x) = \left(\frac{\sin(x)}{\cos(x)}\right)' = \frac{\cos^2(x) + \sin^2(x)}{\cos^2(x)} = \frac{1}{\cos^2(x)} = \sec^2 x \qquad (8)$$

(b) (10 points)

$$f(x) = (x^2 + 3)\sqrt{1 - x^3}$$

Solution:

$$f'(x) = 2x\sqrt{1 - x^3} + (x^2 + 3)\frac{-3x^2}{2\sqrt{1 - x^3}} \qquad (9)$$

6. (10 points) Compute the derivative of the following function using the definition.

$$f(x) = \frac{2}{x+1}$$

Solution:

$$f'(x) = \lim_{h\to 0} \frac{1}{h}\left(\frac{2}{x+h+1} - \frac{2}{x+1}\right) = \lim_{h\to 0} \frac{1}{h}\frac{-2h}{(x+1)(x+h+1)} = \qquad (10)$$

$$= \lim_{h\to 0} \frac{-2}{(x+1)(x+h+1)} = \frac{-2}{(x+1)^2} \qquad (11)$$

7. (10 points) Use implicit differentiation to find the tangent line to $\cos(x)\sin(y) = \frac{1}{4}$ at $(\frac{\pi}{3}, \frac{\pi}{6})$.

Solution: we have:

$$-\sin(x)\sin(y) + \cos(x)\cos(y)\frac{dy}{dx} = 0 \qquad (12)$$

so that

$$\frac{dy}{dx} = \tan(x)\tan(y) \qquad (13)$$

so that the slope m of the tangent line at $(\frac{\pi}{3}, \frac{\pi}{6})$ is 1. The equation of the tangent is thus:

$$y = \frac{\pi}{6} + \left(x - \frac{\pi}{3}\right) = x - \frac{\pi}{6} \qquad (14)$$

8. A bullet is shot straight up with an initial velocity of 160 ft/s. It reaches the height of

$$h(t) = 160t - 16t^2$$

feet after t second.

(a) (7 points) What is the velocity of the bullet when its height is 400ft? Remember that the velocity is the rate of change of the position.

Solution: $h(t) = 400$ implies that $t = 5$. The velocity is

$$v(t) = h'(t) = 160 - 32t. \tag{15}$$

At $t = 5$ we have:

$$h'(5) = 0 \tag{16}$$

(b) (7 points) What is the acceleration of the bullet at any time t? Remember that the acceleration is the rate of change of the velocity.

Solution: The accelaration $a(t) = v'(t) = -32$ for every t.

(c) (10 points) How high does the bullet go?

Solution:
The trajectory is an inverted parabola so that the maximum is at $t = \frac{160}{2 \cdot 16} = 5$. The maximum height is 400ft.