MATH 2401, PRACTICE TEST 3

1) Let Q be the triangle delimited by the lines $x = y$, $x = -y$ and $x = 1$. The triangle Q is occupied by a laminar object L with density $f(x, y) = ye^{-x}$. Compute the total mass M of the object L and its center of mass (x_M, y_M).

2) Compute the integral of the function

$$f(x, y) = \text{atan}\left(\frac{y}{x}\right)(x^2 + y^2)$$

on the domain Q_1 given by the points (x, y) such that $-y \leq x \leq y$ and $x^2 + y^2 \leq 1$.

3) Let $f(x, y, z)$ be a continuous function and Q_2 the domain the domain bounded by the surfaces $z = 0$, $z = y$ and $x^2 = 1 - y$.

a) Express

$$\int\int\int_V f(x, y, z)dxdydz$$

as an iterated integral. How many different way you have to do it?

b) Let

$$f(x, y, z) = z^3 \frac{1}{\sqrt{1 + x^2}}.$$

Can you reduce the above iterated integral to an integral on one variable?

4) Let Q_3 be an object delimited by the surfaces $z = y$, $z = -y$, $z = 1$, $x = 1$ and $x = -1$ and with a density $f(x, y, z) = x^2 ye^{-z}$. Compute the total mass M of Q_3 and its center of mass (x_M, y_M, z_M).

5) Let Q_4 the region bounded by the surfaces $z = x^2 + y^2$ and $z = 1$. Compute the integral of

$$f_1(x, y, z) = (x^2 + y^2)e^{-z}$$
$$f_2(x, y, z) = x(x^2 + y^2)e^{-z}$$
$$f_3(x, y, z) = z(x^2 + y^2)e^{-z}$$

on Q_4.(Hint: use cylindrical coordinates)