

Answer all questions; show all work; closed books, no calculators. THE HONOR CODE APPLIES TO THIS CLASS

Problem	Points	Score
1	50	50
2	50	50
3	50	50
4	50	50
Total	200	200

1. (50pts) Let A , B and C be three independent events which occur with respective probability $\mathbb{P}(A) = 1/4$, $\mathbb{P}(B) = 1/2$ and $\mathbb{P}(C) = 1/3$.

(a) (25pts) Find the probability that at least one of the events occurs.

$$\begin{aligned} \mathbb{P}(\text{at least one}) &= 1 - \mathbb{P}(\text{none}) \\ &= 1 - \mathbb{P}(A^c \cap B^c \cap C^c) \\ \underline{\text{II}} \quad &= 1 - \mathbb{P}(A^c) \mathbb{P}(B^c) \mathbb{P}(C^c) \\ &= 1 - 3/4 \cdot 1/2 \cdot 2/3 = 1 - 1/4 = 3/4 \end{aligned}$$

(b) (25pts) Find the probability that exactly two of the events occur.

$$P(\text{exactly 2}) = P((A^c \cap B \cap C) \cup (A \cap B^c \cap C) \cup (A \cap B \cap C^c))$$

$$\checkmark = P(A^c \cap B \cap C) + P(A \cap B^c \cap C) + P(A \cap B \cap C^c)$$

pairwise disjoint

$$\Downarrow = P(A^c)P(B)P(C) + P(A)P(B^c)P(C) + P(A)P(B)P(C^c)$$

$$= \frac{3}{4} \cdot \frac{1}{2} \cdot \frac{1}{3} + \frac{1}{4}$$

2. (50pts) A medical doctor wishes to detect a particular chemical substance in a blood sample. A test indicates the presence of this chemical substance, when it is there, 90% of the time; however this test also produces false positive and indicates the presence of this chemical substance, when it is not there, 5% of the time. It is known that 80% of the samples do not contain this substance (and 20% do).
- (a) (25pts) What is the probability that for a randomly selected sample, the test is positive, i.e., detects the substance?

(b) (25pts) What is the probability that a randomly selected sample does actually contain the substance, given that the test is positive?

3. (50pts) I throw two fair dice, one four-sided and the other three sided, and record their respective score S_1 and S_2 . Let X be the sum of the scores, i.e., $X = S_1 + S_2$ and let Y be the difference of the scores, i.e., $Y = S_1 - S_2$. Find the joint probability mass function of X and Y . Are X and Y independent?

4. (50pts) The sea floor of a particular Tahitian lagoon contains N oysters, where N is a Poisson random variable with parameter $1/100$, and each oyster contains a black pearl, independently of the other oysters, with fixed probability $1/1000$. Oysters are harvested from the bottom of the lagoon, opened, and let X be the number containing a black pearl.

(a) (20pts) Find $p_{X|N=n}$, the conditional pmf of X given $\{N = n\}$, $n = 0, 1, 2, \dots$

(b) (20pts) Find $\mathbb{E}(X|N)$.

(c) (10pts) Find $\mathbb{E}X$, the expectation of X .