MATH 3235, Test I **Houdré**

Thursday October 11, 2018 **Name: ______________________________**

Answer all questions; show all work; closed books, no calculators. THE HONOR CODE APPLIES TO THIS CLASS

Problem	Points	Score
1	50	
2	50	
3	50	
4	50	
Bonus	20	
Total	200	

1. (a) (25pts) Let A and B be two events which occur with respective probability $\mathbb{P}(A) = 1/4$, $\mathbb{P}(B) = 1/2$ and such that $\mathbb{P}(A \cup B) = 3/5$. Are A and B independent events? Justify your answer.

Since $\mathbb{P}(A \cap B) = \mathbb{P}(A) + \mathbb{P}(B) - \mathbb{P}(A \cup B)$

We have

$$\mathbb{P}(A \cap B) = \frac{1}{4} + \frac{1}{2} - \frac{3}{5} = \frac{3}{4} - \frac{3}{5} = \frac{3}{20}$$

But

$$\mathbb{P}(A)\mathbb{P}(B) = \frac{1}{4} \cdot \frac{1}{2} = \frac{1}{8}$$

Since

$\mathbb{P}(A \cap B) \neq \mathbb{P}(A)\mathbb{P}(B)$, the two events are not $\perp\!\!\!\perp$

they are dependent.

(b) (25pts) Let A, B and C be three events which occur with respective probability $\mathbb{P}(A) = 1/3$, $\mathbb{P}(B) = 1/2$ and $\mathbb{P}(C) = 1/5$. Find the probability that exactly two of the events occur.

Exactly two corresponds to

$$(A^c \cap B \cap C) \cup (A \cap B^c \cap C) \cup (A \cap B \cap C^c)$$

pairwise disjoint

Hence

$$\mathbb{P}(\text{exactly two}) = \mathbb{P}(A^c \cap B \cap C) + \mathbb{P}(A \cap B^c \cap C) + \mathbb{P}(A \cap B \cap C^c)$$

$$= \mathbb{P}(A^c)\mathbb{P}(B)\mathbb{P}(C) + \mathbb{P}(A)\mathbb{P}(B^c) \perp\!\!\!\perp$$

$\perp\!\!\!\perp$

2. (50pts) Consider two urns. The first one has two white balls and seven black balls and the second has seven white balls and six black balls. A fair coin is flipped and if heads comes up a ball is drawn from the first urn while if the coin shows tails then a ball is drawn from the second urn. Given that a white ball has been drawn, what is the probability that the outcome of your coin toss was head?

3. (50pts) I throw two fair dice, the first is three-sided and the second is four-sided, and record their respective score S_1 and S_2. Let X be the product of the scores, i.e., $X = S_1 \times S_2$ and let Y be the quotient of the scores, i.e., $Y = S_1/S_2$. Find the joint probability mass function of X and Y. Are X and Y independent?

4. (50pts) Each morning my grandmother used to go to her garden to collect the eggs laid over the previous night by her hens, in order to sell them to the weekly market. Unfortunately, each time my grand-mother collected an egg there was a $1/100$ change she will break it (and thus the broken eggs could not be sold to the market) and all the breaks are independent of each other. Assume that the number of eggs laid during a week is a Poisson random variable N with parameter $\lambda = 100$. Denote by X the number of eggs my grandmother weekly sells at the market.

   (a) (30pts) Fix $n \in \{0, 1, 2, \ldots\}$, and find $p_{X|N=n}$, the conditional pmf of X given $\{N = n\}$,

(b) (10pts) Find $\mathbb{E}(X|N = n)$.

(c) (10pts) Find $\mathbb{E}X$, the expectation of X.

(d) (Bonus 20 points) Find G_X the probability generating function of X. What do you conclude about X? **Hint: To find G_X, remember the partition theorem.**