7.4 n 6

Since the prior distribution is a Gamma distribution with mean μ_0 we know that $\frac{\alpha}{\beta} = \mu_0$. The posterior distribution will be a Gamma with par. $\alpha' = \alpha + n\bar{X}_n$ and $\beta' = \beta + n$ so that the mean of the posterior distribution is

$$\frac{\alpha'}{\beta'} = \frac{\alpha + n\bar{X}_n}{\beta + n} = \frac{\alpha}{\beta+n} + \frac{n}{\beta+n}\bar{X}_n$$

Calling $\gamma_n = \frac{n}{\beta+n}$ and observing that

$$\frac{\alpha}{\beta+n} = \frac{\alpha}{\beta}\,\frac{\beta}{\beta+n} = \mu_0(1-\gamma_n)$$

we get $\frac{\alpha'}{\beta'} = \gamma_n\bar{X}_n + (1-\gamma_n)\mu_0$

Clearly γ