Fall 04 Name: _____

Math 3770 Final Bonetto

1) Let X_1 and X_2 two discrete random variables with a joint p.m.f given by:

$$p(1,0) = p(-1,0) = p(0,1) = p(0,-1) = 0.25$$

while $p(x_1, x_2) = 0$ in all other cases. Compute:

a) the marginal p.m.f. $p_{X_1}(x_1)$ and the conditional p.m.f. $p_{X_2|X_1}(x_2|x_1)$.

b) $E(X_1)$, $V(X_1)$, $\text{Cov}(X_1, X_2)$. Are X_1 and X_2 independent?

Let now $Y_1 = X_1 + X_2$ and $Y_2 = X_1 - X_2$.

c) Compute the joint p.m.f. of Y_1 and Y_2. Are Y_1 and Y_2 independent? (**Hint** What are the possible value of (Y_1, Y_2)?)

2) The following data are the lifetimes x_i of a sample of $n = 20$ bulbs.

0.008	1.067	0.056	0.121	1.147	0.687	1.960	0.486	0.194	0.192
0.142	2.539	0.235	1.034	0.241	0.130	0.840	0.350	0.337	0.576

You know that $\sum_{i=1}^{20} x_i = 12.34$. Assume that the lifetimes X_i of the bulb are independent r.v. with exponential distribution with parameter λ.

a) Find the MLE for $\hat{\lambda}$ for λ. Compute its value for the above data.

b) Let $Y = \sum_i X_i$. It can be proven that the p.d.f. of Y is

$$f(y) = \frac{\lambda^n}{(n-1)!} y^{n-1} e^{-\lambda y}.$$

Compute $E(\hat{\lambda})$. Is the estimator unbiased? **Hint** use that

$$\int_0^\infty x^n e^{-x} = n!$$

2) (Continued) Suppose now that instead of having the full result of the sample the experimenter tells you only the smallest value observed, *i.e.* you only know $Z = \min_i X_i$.

   c) Compute the p.d.f. $g(z)$ of Z. (**Hint** observe that $Z > z$ if and only if for all i, $X_i > z$)

   d) Find a MLE for λ in this situation. Compute its value for the data reported at the beginning of the exercise.

3) Two components are connected in series as in figure 1.

The lifetime T_1 of the first component is an exponential random variable with parameter 1 while the lifetime T_2 of the second component is an exponential random variable with parameter 2. The system fails if one of the two components fails. Call T_s the lifetime of the system.

a) compute the p.d.f of T_s.

b) If at time t you observe that the system has failed what is the probability that the first component failed?

3) (Continued) The same two components are now connected in parallel like in Figure 2.

The system fails only if both component fail. Call T_p the lifetime of the system.

c) compute the p.d.f of T_p.

b) If at time t you observe that the system is working what is the probability that the first bulb has already failed? and probability that the second bulb is still working?

4) You run a random sample of size $N = 100$ to measure the maximum stress a given type a steel cable can support. After ordering them in increasing order you obtain:

700.0	700.4	700.6	701.8	701.9	702.6	703.1	705.2	707.4	708.2
710.0	711.1	711.8	712.6	712.6	713.4	713.8	714.3	716.3	717.1
717.6	718.6	719.2	719.7	725.3	726.6	727.0	728.7	731.5	733.3
733.4	736.6	736.7	737.0	737.4	737.8	739.2	742.4	742.7	744.2
744.5	744.7	746.4	746.4	746.6	746.8	747.9	749.6	750.9	751.2
752.5	752.5	754.2	756.9	757.7	758.6	758.6	761.0	761.7	762.4
766.7	767.0	767.6	767.8	768.0	768.6	770.1	772.8	772.8	773.8
775.2	775.6	776.9	777.0	777.4	778.3	778.4	778.5	779.9	781.0
781.1	783.0	784.6	785.2	785.3	786.1	786.8	787.1	787.6	788.4
789.3	790.4	790.8	794.0	794.4	795.4	796.4	796.6	796.6	798.4

and you know that:

$$\sum_i x_i = 75126.32 \qquad \sum_i x_i^2 = 56527886.33$$

a) compute the sample average and standard deviation.

d) give a 95% CI for the true population average μ.

b) compute the sample median and fourth spread and draw a box plot.

c) after choosing the number of classes and computing the relative frequency for every class, sketch a histogram for the above data. Do you think the population has a normal distribution? Why?

For your project it is very important that $\mu > 750$. If μ is not strictly greater than 750 you cannot use the cable.

e) formulate a null hypothesis H_0 and an alternative hypotheses H_a. Test this hypotheses at 0.05 significance level.

f) Test this hypotheses at 0.1 significance level.

g) What is the minimum significance level for which you to reject H_0?

5) At the beginning of the year you can buy two possible stocks: stock A and stock B. You know that, every day, the price of stock A has a probability 0.7 to increase of 1\$ and 0.3 of decrease of 1\$ while the price of stock B has a probability 0.54 to increase of 8\$ and 0.46 to decrease of 8\$.

   a) Compute the approximate distribution function of the change of the price of stock A and stock B after 50 days and after 100 days.

   b) Compute the probability that after 100 days the price of stock A increased more than the price of stack B.

   c) You want to be sure at 99% that after 50 days you will not end up loosing money. Would you buy stock B?