Spring 06
Math 3770

Name: ____________________

Final

Bonetto

1a		4a	
1b		4b	
2a		4c	
2b		4d	
3a		4e	
3b		5a	
3c		5b	

1) Let X_1 and X_2 be two discrete random variables with a joint p.m.f given by:

$$p(1,0) = p(-1,0) = p(0,1) = p(0,-1) = 0.1$$
$$p(0,0) = 0.6$$

while $p(x_1, x_2) = 0$ in all other cases. Compute:

a) the marginal p.m.f. $p_{X_1}(x_1)$ and the conditional p.m.f. $p_{X_2|X_1}(x_2|x_1)$.

b) $E(X_1)$, $V(X_1)$, $\mathrm{Cov}(X_1, X_2)$. Are X_1 and X_2 independent?

2) Let X and Y be two continuous r.v. with joint p.d.f given by:

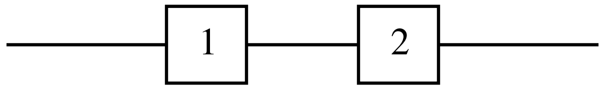
$$f(x,y) = \begin{cases} \frac{1}{4} & \text{if } 0 < x < 2 \text{ and } 0 < y < 2 \\ 0 & \text{otherwise} \end{cases}.$$

Let $A = \{X < Y\}$ and $B = \{X < 1\}$ be two events.

a) Compute $P(A) = P(X < Y)$ and $P(B) = P(X < 1)$.

b) Compute the conditional probability that $X < Y$ given that $X < 1$, that is $P(A|B)$. Are A and B independent?

3) Two components are connected in series as in the figure below.



The lifetime T_1 of the first component is an exponential random variable with parameter λ while the lifetime T_2 of the second component is an exponential random variable with parameter 2λ. The two lifetimes are independent. The system fails when one of the two the components fail. Call T_p the lifetime of the system.

a) Compute the p.d.f $f(t)$ of T_p. (**Hint:** Compute first the c.d.f. $F(t) = 1 - P(T_p > t)$ by writing the condition $T_p > t$ in term of T_1 and T_2.)

b) If at time t you observe that the system has already failed what is the probability that the failure was due to the first component?

(Continued) The following data are the lifetimes t_i, in months, of a random sample of $n = 20$ systems like the one described above:

0.3532 7.8604 0.7676 1.9379 0.3683 1.3751 0.6865 1.6931 2.3774 3.3251
3.2857 0.7648 2.3896 1.2262 3.7826 0.7194 1.9437 0.3016 0.3165 1.0432.

You know that $\sum_{i=1}^{20} t_i = 36.52$.

c) Derive the MLE $\hat{\lambda}$ for λ (show your work). Compute its value for the above data.

4) You run a random sample of size $N = 100$ to measure the maximum stress a given type a steel cable can support. After ordering the data in increasing order you obtain:

4.011	4.018	4.026	4.029	4.036	4.076	4.077	4.090	4.111	4.131
4.177	4.192	4.227	4.233	4.233	4.245	4.272	4.293	4.314	4.317
4.319	4.324	4.336	4.345	4.384	4.400	4.424	4.424	4.430	4.439
4.454	4.468	4.479	4.485	4.485	4.531	4.535	4.544	4.581	4.624
4.641	4.647	4.656	4.679	4.689	4.702	4.721	4.740	4.770	4.794
4.801	4.831	4.835	4.851	4.867	4.870	4.883	4.897	4.930	4.938
4.947	4.994	5.038	5.040	5.060	5.077	5.086	5.097	5.106	5.112
5.118	5.121	5.133	5.164	5.184	5.184	5.189	5.208	5.216	5.229
5.232	5.245	5.263	5.274	5.279	5.291	5.296	5.304	5.330	5.339
5.341	5.352	5.405	5.416	5.435	5.456	5.466	5.471	5.478	5.492

and you know that:

$$\sum_i x_i = 477.38 \qquad \sum_i x_i^2 = 2299$$

a) compute the sample average and standard deviation.

b) give a 95% CI for the true population average μ ($z_{0.025} = 1.96$).

c) compute the sample median and fourth spread and draw a box plot.

d) after choosing the number of classes and computing the relative frequency for every class, sketch a histogram for the above data.

e) You want a 95% CI with precision better than 0.1. This means that you want an interval of the form $[\bar{x} - w, \bar{x} + w]$ with $w < 0.05$. How big should your sample be to obtain such a precision?

5) At the beginning of the year you can buy two possible stocks: stock A and stock B. You know that, every day, the price of stock A has a probability 0.7 to increase of 2\$ and 0.3 to decrease of 2\$ while the price of stock B has a probability 0.6 to increase of 5\$ and 0.4 to decrease of 5\$. Stock A and B change price independently from each other.

   a) Compute the approximate distribution function of the change of the price of stock A and stock B after 50 days.

   b) Compute the probability that after 50 days the price of stock A increased more than the price of stack B. Express it in term of the function Φ.