1) In a box there are 3 red balls and 3 blue balls. You extract a ball at random. Let X be the random variable that describe the result of this extraction with $X = 1$ if the ball extracted is red and $X = 0$ if it is blue. Without reinserting the first ball you extract a second ball. Let Y be the random variable that describe the result of this second extraction with $Y = 1$ if the ball extracted is red and $Y = 0$ if it is blue.

a) find the joint p.d.f. of X and Y.
b) find the marginal p.d.f of X and of Y.
c) find $E(X)$ and $E(Y)$.
d) find the conditional distribution function of X given Y.
e) find $\mathrm{Cov}(X, Y)$.
f) are X and Y independent? Explain.

2) In a town with 1000000 voters there are two parties, party A and party B. You know that 55% of the voters prefer party A and 45% prefer party B. When next election come not all voters will vote. You know that the probability that a voter that prefer party A will vote is 0.7 and the probability that a voter that prefer party B will vote is 0.9.

a) given a randomly chosen voter, what is the probability that he/she will vote?
b) given a randomly chosen voter, what is the probability that he/she will vote and vote for party A.
c) given a randomly chosen voter, what is the probability that he/she will vote and vote for party B.
Let S_A be the random variable describing the number of voters that will vote and vote for party A and S_B be the random variable describing the number of voters that will vote and vote for party B. Assume that every voter will decide whether to vote or not independently from the other voters.
d) write an approximate p.d.f. for S_A, S_B and $S_A - S_B$. (**Hint**: Let X_i that r.v. that describe whether a voter that prefer party A willvote or not, *i.e.* $X_i = 1$ if he votes and $X_i = 0$ if he doesn't. Write S_A in term of the X_i.)
e) What is the probability that party A will win the election.

3) Let X de a discrete r.v. with p.m.f

$$f(i) = \begin{cases} \dfrac{\lambda^i}{1-\lambda} & \text{if } i \geq 0 \\ 0 & \text{if } i < 0 \end{cases}$$

Find a maximum likelihood estimator for λ. Is it unbaised? Explain.

4) Let X and Y be two r.v. with joint p.m.f given by

$$f(x, y) = \begin{cases} \frac{1}{2} & \text{if } 0 < x < 1 \text{ and } 0 < y < 1 \\ \frac{1}{2} & \text{if } -1 < x < 0 \text{ and } -1 < y < 0 \\ 0 & \text{otherwise} \end{cases}$$

a) find the marginal p.d.f. of X and Y.
b) are X and Y independent? Explain.

5) Let X be a r.v. uniformly distributed between 0 and 1. Write the p.d.f. of $Y = 2X + 1$.