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a) Prob exactly 10 arrives.

$$e^{-\alpha\tau}\frac{(\alpha\tau)^{10}}{10!} = e^{-10}\frac{10^{10}}{10!} = P_1$$

Prob that all 10 have no violation

$$2^{-10} = P_2$$

The prob. is

$$P = P_1 \cdot P_2 = 2^{-10} e^{-10}\frac{10^{10}}{10!}$$

b) Prob exactly y arrive

$$e^{-10}\frac{10^{y}}{y!} = P_1$$

Prob that 10 have no violation

$$\binom{y}{10} 2^{-y}\, 2^{-(y-10)} = \binom{y}{10} 2^{-y} P_2$$

The prob is

$$P(y) = P_1 P_2 = \binom{y}{10} e^{-10}\frac{10^{y}}{y!}\cdot 2^{-y} =$$

$$= \frac{1}{10!\,(y-10)!} \; 5^{y} \, e^{-10}$$

c) The prob is

$$\sum_{y=10}^{\infty} \frac{1}{10!\,(y-10)!} \, e^{-10} \, 5^{y} =$$

$$\frac{1}{10!} \, e^{-10} \, 5^{10} \sum_{y=10}^{\infty} \frac{1}{(y-10)!} \, 5^{y-10} =$$

$$\frac{e^{-10} \, 5^{10}}{10!} \sum_{y=0}^{\infty} \frac{1}{y!} \, 5^{y} = \frac{e^{-10} \, 5^{10} \, e^{5}}{10!} =$$

$$= \frac{5^{10} \, e^{-5}}{10!}$$

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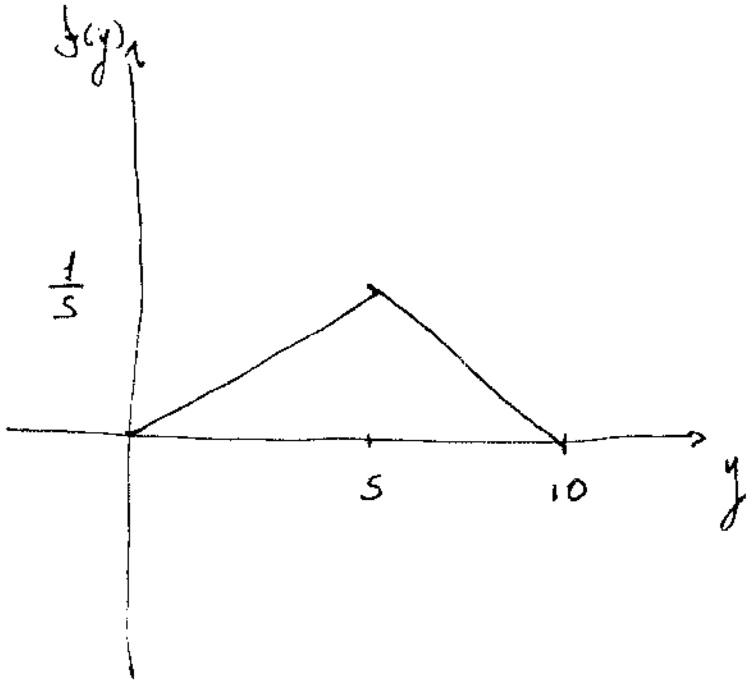
a)



b) $\int_0^5 \frac{1}{25} y \, dy = \frac{1}{25} \frac{y^2}{2} \Big|_0^5 = \frac{1}{2}$

$\int_5^{10} \left(-\frac{1}{25} y + \frac{2}{5}\right) dy = -\frac{1}{25} \frac{y^2}{2} \Big|_5^{10} + \frac{2}{5} y \Big|_5^{10} = \frac{1}{2}$

c) $P(\text{waiting Time} < 3) = \int_0^3 f(y) \, dy =$

$\int_0^3 \frac{1}{25} y \, dy = \frac{1}{25} \frac{y^2}{2} \Big|_0^3 =$

$\frac{1}{25} \cdot \frac{9}{2}$

d) $P(\text{waiting Time} < 8) = 1 - P(\text{waiting Time} > 8) =$

$$1 - \int_8^{10}\left(\frac{2}{5} - \frac{1}{25}y\right)dy = 1 - \frac{2}{5}y\Big|_8^{10} + \frac{1}{25}\frac{y^2}{2}\Big|_8^{10} =$$

$$= 1 - \frac{4}{5} + \frac{1}{25}\cdot 18 = 1 - \frac{2}{25}$$

e) $P(3 \le \text{waiting Time} \le 8) =$

$P(\text{waiting Time} \le 8) - P(\text{waiting Time} \le 3) =$

$$1 - \frac{2}{25} - \frac{9}{50} = \frac{50 - 4 - 9}{50} = \frac{37}{50}$$

f) $P(\text{waiting Time} < 2 \text{ or } > 6) =$

$P(\text{waiting Time} < 2) + P(\text{witing Time} > 6) =$

$$\int_0^2 \frac{1}{25}y\,dy + \int_6^{10}\left(\frac{2}{5} - \frac{1}{25}y\right)dy =$$

$$\frac{1}{25}\frac{y^2}{2}\Big|_0^2 + \frac{2}{5}y\Big|_6^{10} - \frac{1}{25}\frac{y^2}{2}\Big|_6^{10} =$$

$$\frac{2}{25} + \frac{8}{5} - \frac{1}{25}32 = \frac{2 + 40 - 32}{25} = \frac{10}{25} = \frac{2}{5}$$

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$$f(x) = \begin{cases} \frac{1}{B-A} & A \le x \le B \\ 0 & \text{otherwise} \end{cases} \qquad \text{p.d.f.}$$

$$F(x) = \begin{cases} 0 & x \le A \\ \frac{1}{B-A}(x-A) & A \le x \le B \\ 1 & x \ge B \end{cases}$$

$\eta(p) = 100p$ percentile satisfy

$$F(\eta(p)) = p$$

$$\frac{1}{B-A}(\eta(p) - A) = p$$

$\eta(p) = (B-A)p + A = 100p$ percentile

b) $$E(X) = \int_A^B \frac{1}{B-A}\, x\, dx = \frac{1}{B-A} \left.\frac{x^2}{2}\right|_A^B =$$

$$= \frac{B^2 - A^2}{2(B-A)} = \frac{B+A}{2}$$

$$V(X) = E(X^2) - E(X)^2$$

$$E(X^2) = \int_A^B \frac{x^2}{B-A}\,dx = \frac{1}{B-A}\,\frac{x^3}{3}\Big|_A^B =$$

$$\frac{1}{3}\,\frac{B^3 - A^3}{B - A} = \frac{1}{3}\left(A^2 + AB + B^2\right)$$

$$E(X)^2 = \frac{(A+B)^2}{4}$$

$$\cancel{Var(X) = \frac{1}{6}\left(2A^2 + 2AB + 2B^2 - 3(A+B)^2\right) =}$$

$$Var(X) = \frac{1}{12}\left(4A^2 + 4AB + 4B^2 - 3(A+B)^2\right) =$$

$$\frac{1}{12}\left(A^2 + B^2 - 2AB\right) = \frac{1}{12}(A - B)^2$$

$$\sigma_X = \frac{1}{2\sqrt{3}}\,|A - B|$$

c) $$E(X^n) = \int_A^B \frac{x^n}{B-A} = \frac{1}{B-A}\,\frac{X^{n+1}}{n+1}\Big|_A^B = \frac{1}{n+1}\,\frac{B^{n+1} - A^{n+1}}{B - A},$$