If x_i, $i = 1 \dots N$, are data from a sample then:

$$\bar{x} = \frac{1}{N}\sum_{i=1}^{N} x_i$$

$$\sigma_x^2 = \frac{1}{N-1}\left(\sum_{i=1}^{N} x_i^2 - N\bar{x}^2\right)$$

$$\tilde{x} = \begin{cases} x_{\frac{N+1}{2}} & N \text{ odd} \\ \frac{x_{\frac{N}{2}} + x_{\frac{N}{2}+1}}{2} & N \text{ even} \end{cases}$$

Probabilities:

$$P(\mathcal{S}) = 1 \qquad 0 \le P(A) \le 1 \qquad A \cap B = \emptyset \Rightarrow P(A \cup B) = P(A) + P(B)$$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

$$A \text{ and } B \text{ are independent } \Leftrightarrow P(A \cap B) = P(A)P(B)$$

$$P(B) = P(B|A)P(A) + P(B|A^c)P(A^c)$$

$$P(A|B) = \frac{P(B|A)P(A)}{P(B|A)P(A) + P(B|A^c)P(A^c)}$$

Random Variable: discrete

$$p(x) = P(X = x) \qquad F(x) = P(X \le x) = \sum_{y \le x} p(y)$$

$$E(X) = \mu_X = \sum_x xp(x) \qquad E(h(x)) = \sum_x h(x)p(x)$$

$$p(x, y) = P(X = x \;\&\; Y = y) \qquad p_X(x) = \sum_y p(x, y)$$

$$p_{X|Y}(x|y) = \frac{p(x, y)}{p_Y(y)} \qquad E(h(X, Y)) = \sum_x h(x, y)p(x, y)$$

Random Variable: continuous

$$P(a \le X \le b) = \int_a^b f(x)dx \qquad F(x) = P(X \le x) = \int_{y \le x} f(y)dy$$

$$E(X) = \mu_X = \int_{-\infty}^{\infty} xf(x)dx$$

The 100p percentile x_p is given by$F(x_p) = p$

$$P((X,Y)\in A)=\int_A f(x,y)dxdy \qquad f_X(x)=\int_{-\infty}^{\infty} f(x,y)dy \qquad f_{X|Y}(x|y)=\frac{f(x,y)}{f_Y(y)}$$

$$E(h(X))=\int_{-\infty}^{\infty} h(x)f(x)dx \qquad E(h(X,Y))=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} h(x,y)f(x,y)dxdy$$

Variances

$$V(X)=\sigma_X^2=E(X^2)-E(X)^2 \qquad Cov(X,Y)=E(XY)-E(X)E(Y)$$

$$Corr(X,Y)=\rho_{X,Y}=\frac{Cov(X,Y)}{\sigma_X\sigma_Y}$$

Independence: X,Y are independent if and only if $p(x,y)=p_X(x)p_Y(y)$ $(f(x,y)=f_X(x)f_Y(y))$. If X,Y are independent $Cov(X,Y)=0$.

Linear combination

$$E(aX+b)=aE(X)+b \qquad V(aX+b)=a^2V(X)$$

$$E(aX+bY)=aE(X)+bE(Y) \qquad V(aX+bY)=a^2V(X)+b^2V(Y)+2Cov(X,Y)$$

If X_i, $i=1\dots N$, are independent and identically distributed and $\bar{X}=1/N\sum_{i=1}^{N}X_i$ then $\mu_{\bar{X}}=\mu_X$ and $V(\bar{X})=V(X)/N$.

Propagation of error: if U is a function of X and σ_X is small

$$\sigma_U=\left|\frac{dU}{dX}\right|\sigma_X$$

if U is a function of $X,X_2,\dots X_n$ and σ_{X_i} are small

$$\sigma_U=\sqrt{\left(\frac{dU}{dX_1}\right)^2\sigma_{X_1}+\left(\frac{dU}{dX_2}\right)^2\sigma_{X_2}+\cdots+\left(\frac{dU}{dX_n}\right)^2\sigma_{X_n}}$$