

Spring 04
Math 3770

Name: _____
Quiz 1 Bonetto

A University gives four possible degrees: Mathematics, Physics, Biology and Economics. At the end of last year 600 student graduated of which 60 in Mathematics, 120 in Physics, 240 in Biology and 180 in Economics. Advanced Calculus is one of the exams offered by the University. This exam was passed by 90% of the students graduated in Mathematics, 75% of those graduated in Physics, 20% of those graduated in Biology and 30% of those graduated in Economics. Given that a randomly chosen student passed Advanced Calculus what is the probability that he/she was graduated in Biology or Economics.

(Hint: call A_1, A_2, A_3, A_4 the events $\{\text{student graduated in Mathematics}\}$, $\{\text{student graduated in Physics}\}$, $\{\text{student graduated in Biology}\}$, $\{\text{student graduated in Economics}\}$, respectively, and B the event $\{\text{student passed Advanced Calculus}\}$. Write the informations given by the text as probabilities or conditional probabilities of events. Write the question of the exercise as a conditional probability. Use Bayes formula or the definition of conditional probability to find the answer. Alternatively you can solve the exercise by counting the number of students in the different events or intersections of events)

Form the text we obtain that

$$P(A_1) = \frac{60}{600} = 0.1 \quad P(A_2) = \frac{120}{600} = 0.2 \quad P(A_3) = \frac{240}{600} = 0.4 \quad P(A_4) = \frac{180}{600} = 0.3$$

and

$$P(B|A_1) = 0.9 \quad P(B|A_2) = 0.75 \quad P(B|A_3) = 0.2 \quad P(B|A_4) = 0.3$$

The exercise requires to find

$$P(A_3 \cup A_4|B) = \frac{P((A_3 \cup A_4) \cap B)}{P(B)} = \frac{P(A_3 \cap B)}{P(B)} + \frac{P(A_4 \cap B)}{P(B)}$$

This is true because $(A_3 \cup A_4) \cap B = (A_3 \cap B) \cup (A_4 \cap B)$ and $(A_3 \cap B) \cap (A_4 \cap B) = \emptyset$.

We have

$$P(B) = \sum_{i=1}^4 P(B|A_i)P(A_i) = 0.41$$

and

$$P(A_3 \cap B) = P(B|A_3)P(A_3) = 0.08 \quad P(A_4 \cap B) = P(B|A_4)P(A_4) = 0.09$$

Substituting in the above equation we get $P(A_3 \cup A_4|B) = 0.4146$

Differently we can observe that we have $0.9 \cdot 60 = 54$ graduated in Mathematics that passed Advanced Calculus, $0.75 \cdot 120 = 90$ graduated in Physics, $0.2 \cdot 240 = 48$ graduated in Biology and $0.3 \cdot 180 = 54$ graduated in Economics. So you have a total of $54 + 90 + 48 + 54 = 246$ graduated that passed Advanced Calculus. Of these $48 + 54 = 102$ were students of Biology or Economics. So that the requested probability is $P(A_3 \cup A_4|B) = 102/246 = 0.4146$