1) After running a sample of size $n = 50$ a researcher found the following values:

0.000	0.001	0.042	0.177	0.365	0.091	0.092	0.487	0.527	0.454
0.233	0.831	0.932	0.568	0.556	0.051	0.767	0.019	0.252	0.298
0.876	0.532	0.920	0.515	0.810	0.188	0.886	0.571	0.077	0.815
0.985	0.118	0.894	0.784	0.101	0.253	0.020	0.378	0.679	0.681
0.753	0.006	0.624	0.126	0.618	0.771	0.187	0.497	0.510	0.316

The same values ordered in increasing order are:

0.000	0.001	0.006	0.019	0.020	0.042	0.051	0.077	0.091	0.092
0.101	0.118	0.126	0.177	0.187	0.188	0.233	0.252	0.253	0.298
0.316	0.365	0.378	0.454	0.487	0.497	0.510	0.515	0.527	0.532
0.556	0.568	0.571	0.618	0.624	0.679	0.681	0.753	0.767	0.771
0.784	0.810	0.815	0.831	0.876	0.886	0.894	0.920	0.932	0.985

a) Knowing that $\sum_{i=1}^{50} x_i = 22.236$ and $\sum_{i=1}^{50} x_i^2 = 14.660$, compute the sample average $\bar{x}$ and sample variance s^2.
b) Compute the median $\tilde{x}$ and the fourth spread f_s.

Our researcher wants to draw an histogram of the data.

c) How may classesw is will he chose?
d) Write the intervals the define these classes.
e) Compute the value of the histogram for each class.

Assume that the numbers come from a population characterized by a uniform distribution between 0 and θ.

f) Give a non biased estimate of θ.
g) Derive a 95% CI for θ based on the above sample. (**Hint**: use the estimator $\hat{\theta}_1 = 2\bar{x}$.)
h) **Bonus** Use the estimator $\hat{\theta}_2 = \frac{n+1}{n}\max(x_i)$ to derive a 95% upper confidence bound for θ.

2) The time between two successive arrivals of a bus at a bus stop is assumed to be distributed exponentially with parameter λ. An actual observation of a sample of size 10 gives the following numbers (in hours):

0.01 2.13 0.11 0.24 2.29 1.37 3.92 0.97 0.38 0.38

Let X_i, $i = 1, \ldots, 10$ be a random sample for this problem.

a) Write the p.d.f. of the X_i.
b) Write the joint p.d.f. of the random sample X_i, $i = 1, \ldots, 10$.
c) Find a MLE $\hat{\lambda}$ estimator for λ.
d) What value of λ you get for the above sample?
e) Is $\hat{\lambda}$ unbiased? If not, can you make it unbiased?

3) Flipping a coin 10000 times you get the following result: 5100 Head and 4900 Tail.

a) Estimate the probability of observing a Head flipping the coin.
b) Give a 99% CI of the probability of observing a Head.
c) In your opinion, is the coin fair?

4) A researcher run a small sample of size $n = 100$ on a given population and obtain a sample average $\bar{x} = 1$ and a sample variance $s^2 = 2$. He wants a 95% CI on the average of the population with a precision of 0.001. Is the above sample large enough? If he has to run a new sample, which size will he choose?