Spring 07 | Name: ______________________________

Math 3770 | Test 2 | Bonetto

1) Let X_1 and X_2 two discrete random variables with a joint p.m.f given by:

$$p(1,0) = p(-1,0) = p(0,1) = p(0,-1) = 0.125$$
$$p(0,0) = 0.5$$

while $p(x_1, x_2) = 0$ in all other cases. Compute:

a) the marginal p.m.f. $p_{X_1}(x_1)$ and the conditional p.m.f. $p_{X_2|X_1}(x_2|x_1)$.

b) $E(X_1)$, $V(X_1)$, $\text{Cov}(X_1, X_2)$. Are X_1 and X_2 independent?

2) The following data are the lifetimes x_i, in months, of a sample of $n = 20$ bulbs.

0.136 2.660 3.661 0.272 2.872 0.874 0.067 0.816 0.277 9.224
3.975 0.399 1.687 0.232 0.616 0.749 2.122 3.591 3.720 3.309

You know that $\sum_{i=1}^{20} x_i = 41.27$. Assume that the lifetimes X_i of the bulbs are independent r.v. with exponential distribution with parameter λ.

a) Find the MLE $\hat{\lambda}$ for λ. Compute its value for the above data.

b) (**Bonus**: to be attempted only after having solved all the rest of the test. The continuation of this problem on the following page does not depend on this point.) Let $Y = \sum_i X_i$. It can be proven that the p.d.f. of Y is

$$f(y) = \frac{\lambda^n}{(n-1)!} y^{n-1} e^{-\lambda y}.$$

Compute $E(\hat{\lambda})$. Is the estimator unbiased? **Hint** use that

$$\int_0^\infty x^n e^{-x} = n!$$

2) (Continued) Suppose now that, instead of obeserving the lifetimes, you switch on the bulbs, come back after one month and observe how many bulbs are still working. Let N be the random variable that describe the number of bulbs still working after one month. Assume, as before, that the lifetimes of the bulbs are independent r.v. with exponential distribution with parameter λ.

   c) compute the p.m.f of N. (**Hint**: each bulb is either working or not working indipendently from the others.)

   d) Use the above results to find an estimator for λ. You can use either ML or the method of moments. Estimate λ if $N = 9$.

3) You run a random sample of size $N = 100$ to measure the maximum stress that a given type a steel cable can support. After ordering the data in increasing order you obtain:

15.65	16.25	16.66	16.79	16.81	17.04	17.11	17.32	17.59	17.70
17.88	17.97	17.98	17.98	18.01	18.10	18.15	18.33	18.35	18.38
18.51	18.61	18.71	18.72	18.82	18.88	19.13	19.20	19.24	19.29
19.36	19.40	19.46	19.48	19.53	19.62	19.68	19.72	19.73	19.80
19.88	19.90	19.92	19.96	19.96	20.00	20.08	20.11	20.17	20.22
20.23	20.25	20.34	20.42	20.43	20.47	20.51	20.57	20.58	20.62
20.65	20.76	20.77	20.85	20.88	21.01	21.05	21.13	21.13	21.14
21.15	21.18	21.22	21.24	21.33	21.35	21.38	21.42	21.42	21.45
21.46	21.56	21.59	21.63	21.71	21.79	21.88	22.13	22.16	22.19
22.37	22.38	22.38	22.50	22.58	22.83	22.88	23.24	23.38	24.36

and you know that:

$$\sum_i x_i = 2009.6 \qquad \sum_i x_i^2 = 40683$$

a) compute the sample average and standard deviation.

b) give a 95% CI for the true population average μ (remember that $z_{0.025} = 1.96$).

c) You want a 95% CI with precision better than 0.2. This means that you want an interval of the form $[\bar{x} - w, \bar{x} + w]$ with $w < 0.1$. How large should your sample be to obtain such a precision?

4) At the beginning of the year you can buy two possible stocks: stock A and stock B. You know that, every day, the price of stock A has a probability 0.7 to increase of 1\$ and 0.3 of decrease of 1\$ while the price of stock B has a probability 0.54 to increase of 8\$ and 0.46 to decrease of 8\$. Stock A and B change price independently from each other.

   a) Use the CLT to obtain the approximate distribution function of the change of the price of stock A and stock B after 50 days.

   b) Compute the probability that after 50 days the price of stock A increased more than the price of stock B. Express it in term of the function Φ.