No books or notes. No cellphone or woreless devices. Write clearly and show your work for every answer.

Name: ______________________________

Question:	1	2	3	Total
Points:	40	30	30	100
Score:				

1. The following number form a sample of size 15 randomly selected from a large population:

3.0337	3.5437	2.5573	2.3133	4.4635
2.9750	5.2821	2.9983	4.7853	5.0585
5.1927	5.4394	3.3518	5.4513	3.0764

(a) (10 points) Compute the median and forth spread and find eventual outliers.

Solution: By sorting the data we get

2.3133	2.5573	2.9750	2.9983	3.0337
3.0764	3.3518	3.5437	4.4635	4.7853
5.0585	5.1927	5.2821	5.4394	5.4513

so that we have

$$\tilde{x} = 3.5437$$
$$lf = \frac{2.9983 + 3.0337}{2} = 3.016 \qquad uf = \frac{5.0585 + 5.1927}{2} = 5.126$$
$$fs = 5.126 - 3.016 = 2.11$$

It is easy to see that there are no outliers.

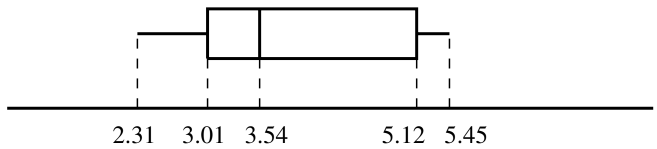
(b) (10 points) Knowing that $\sum_{i=1}^{15} x_i = 59.522$ and $\sum_{i=1}^{15} x_i^2 = 254.77$ compute the sample average and standard deviation.

Solution: We have

$$\bar{x} = \frac{59.522}{15} = 3.97 \qquad s = \sqrt{\frac{1}{14}\left(254.77 - \frac{59.522^2}{15}\right)} = 1.15$$

(c) (10 points) Draw a box plot for the data.

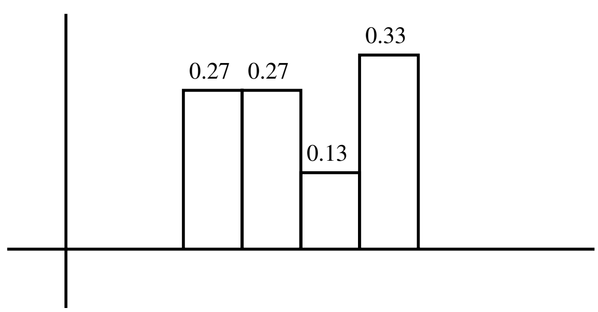
Solution:



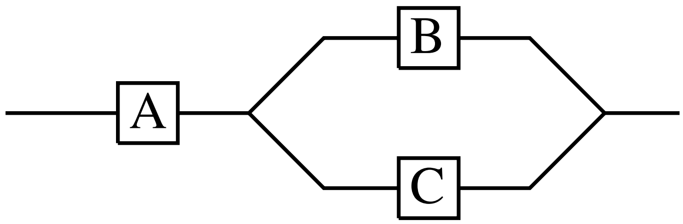
(d) (10 points) Sketch an histogram of the data.

Solution: We choose 4 classes and we get

class	range	freq.	rel freq.	den.
1	2-3	4	0.27	0.27
2	3-4	4	0.27	0.27
3	4-5	2	0.13	0.13
4	5-6	5	0.33	0.33



2. Consider the circuit depicted in the figure.



You know that the entire circuit work if and only if element A and at least one between elements B and C work. Assume that the probability that A works is 0.9 while the probability that B works is 0.7 and the probability that C works is 0.7. Each element works or not independently from the others.

(a) (10 points) Compute the probability that the entire circuit works.

Solution: Call A, B and C the events that element A, B or C (respectively) works. Call I the event the the entire circuit works. We have

$$P(I) = P(A)P(B \cup C) = 0.9P(B \cup C)$$

Observe that

$$P(B \cup C) = 1 - P(B' \cap C') = 1 - P(B')P(C') = \\ = 1 - (1 - P(B))(1 - P(C)) = 1 - 0.3 \cdot 0.3$$

Thus we have

$$P(I) = 0.9(1 - 0.3^2) = 0.819$$

(b) (10 points) Assume that B does not work, what is the probability that the entire circuit works.

Solution: In this case we have

$$P(I|B') = \frac{P([A \cap (B \cup C)] \cap B')}{P(B')} = \frac{P(A \cap C \cap B')}{P(B')} = P(A \cap C) = 0.9 \cdot 0.7 = 0.63$$

(c) (10 points) What is the probability that element A does not work given that the circuit does not work? What is the probability that element B does not work given that the circuit does not work?

Solution: We have

$$P(A'|I') = \frac{P(I' \cap A')}{P(I')} = \frac{1 - P(A)}{1 - P(I)} = \frac{0.1}{1 - 0.9(1 - 0.3^2)} = 0.552$$

and

$$P(B'|I') = \frac{P(I' \cap B')}{P(I')} = \frac{P(I'|B')P(B')}{P(I')} =$$
$$= \frac{(1 - P(I|B'))(1 - P(B'))}{1 - P(I)} = \frac{(1 - 0.9 \cdot 0.7) \cdot 0.3}{1 - 0.9(1 - 0.3^2)} = 0.613$$

3. You repeat 100 times an experiment that has a probability of 0.01 to succeed. Let X the r.v. that describes the number of successes you obtain.

   (a) (10 points) Find the expected value $E(X)$ and the variance $V(X)$ of X.

Solution: X is Binomial with parametern $p = 0.01$ and $n = 100$ so that we have

$$E(X) = np = 1 \qquad V(X) = np(1-p) = 100 \cdot 0.01 \cdot 0.99 = 0.99$$

   (b) (10 points) Compute the probability that $X = 1$ and the probability that $X = 2$.

Solution: We have

$$P(X=1) = \binom{100}{1} 0.01 \cdot 0.99^{99} = 0.99^{99}$$

while

$$P(X=2) = \binom{100}{2} 0.01^2 \cdot 0.99^{98} = \frac{1}{2} 0.99^{99}$$

(c) (10 points) Call Y the r.v. that gives the position of the first success in the sequence of trials. Compute the average of Y given that $X = 1$. (**Hint**: if you know that $X = 1$, i.e. there is only one success, what is the probability that this is the first trial, or the second, or ...)

Solution: Clearly once you know there is only one success, this can be ina position with equal probability. Thus we have

$$E(Y) = 50.5$$