Consider a system of 10 particles in a box $\Lambda = [0, 100] \cap \mathbb{Z}$. This means that the position of particle i is an integer x_i with $0 \leq x_i \leq 100$. To every particle is associated an Poisson process of parameter 1. The Poisson processes of different particle are independent. This means that for every particle you have, at the time 0, an exponential time of parameter 1. These time are independent. Let t_i the time associated to particle i. Particle i does nothing till time t_i. When time t_i arrives particle i jumps. The rule for a jump are the following:

1) chose a direction at random with equal probability.
2) if the site close to the position of the particle in the choosen direction is occupied by another particle or if it is outside the system do nothing.
3) if it is free and inside the system move the particle to that site.

After particle i jumps, a new exponential random time t_i is choosen and the particle waits again for this new length of time.

Start the system with all particle in the central positions of the systems, *i.e.* the sites from 45 to 54 are occupied and all the other are free. Run the system till a total time $T = 10000$. At every time multiple of 100 record the state of the system. This means that you want to know which position are occupied and which one are free.

Repeat the above procedure a large number of times ($\simeq 1000$) and make an histogram of the probability of observing a particle at position x at time $n * 100$ for $n = 1, \ldots, 100$.

You can simulate the above process in two different ways. The first by directly coding the above description. The second in as follow. Chose an random time t with exponential distribution with parameter 10 remeber that this implies that its expected value is $1/10$). wait till time t and then chose one of the particle at random with equal probability. Use the above rules to jumps the choosen particle. Chose a new exponential time and repeat the procedure.

Explain why this methods is equivalent to the previous one.

Use this new procedure to redo the above simulation (same initial condition and same time length and sampling times). Compare the results.

Observe that in this last procedure the times t play a very minor role. Change it by chosing always the time equal to $1/10$ instead than exponential and repeat the simulation. Compare the results with previous results. Explain what you expect to see and why?

In the last example you have simulated a real Markov chain. Describe the transition matrix for this Markov chain and write it explicitley for a system of 2 particle in $\Lambda = [0, 4] \cap \mathbb{Z}$.