1) Consider the equation

$$\dot{x} = \sin x + r$$

where r is a constant.

Identify the fixed point as a function of r and draw the bifurcation diagram.

2) Let A be the 2×2 matrix

$$A = \begin{pmatrix} \alpha & \beta \\ -\beta & \alpha \end{pmatrix}$$

Compute e^{At}. ~~Using the power~~

~~se~~ Use that $A = \alpha I + \beta J$ where

$$J = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}.$$

3) Consider The equation:

$$X = AX$$

with

$$A = \begin{pmatrix} 1-a & a \\ -a & 1+a \end{pmatrix}$$

Find The general solution as a function of a. ~~When is There a bifurcation? Draw The solution for a before, at and after The bifurcation.~~

Let $X_a(t)$ The solution of (1) starting at $X_a(0) = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$. Show That X_a is continuous in a for fixed T.

Consider The ~~pendulum~~ mechanical system

$$\ddot{x} = -V'(x)$$

~~Ass~~ Show That

$$H(x, \dot{x}) = \frac{\dot{x}^2}{2} + V(x)$$

is conserved.

Write The ~~Th~~ first order system of eq relative To This system.

Find The fixed point and discuss Their linearazation

Assume That

$$V(x) = x^4 + a x^2$$

Find The fixed point as a function of a. Is There a bifurcation?

Describe iT.

If $a=0$ ~~given any~~ show that all
The orbit are periodic.
(Bonus) Give a formula for The period.

5) Consider The differential equation

$$\dot{X} = AX$$

with

$$A = \begin{pmatrix} 1 & a \\ a & 1 \end{pmatrix}$$

For which value of a is This syste conjugate To The one defined by The matrix

$$B = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

or

$$C = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

(Bonus) Write The conjugation φ.