Fall 05 Name: ______________________________

Math 4581 Test 1 Bonetto

1) The motion of a pendulum is described by the following equation:

$$\ddot{x}(t) + 8t\dot{x}(t) + 4(5 + 4t^2)x(t) = \exp(-2t^2)\sin(t)$$

a) find the general solution for the equation. (**Hint**: try the substitution $x(t) = \exp(-2t^2)y(t)$.)

b) You want to solve the equation with boundary conditions

$$x(0) = 0 \qquad \dot{x}(\pi) = 0.$$

Find the solution.

c) **(Bonus)** Write the Green function for the boundary conditions of point b). (**Hint**: You wrote an equation for $y(t)$. Compute the Green function for $y(t)$ and ...)

2) Compute the Foureir series of the function:

$$f(x) = \begin{cases} -x & -1 \le x \le 0 \\ 3x & 0 \le x \le 1 \end{cases}$$

You may use that for $-1 \le x \le 1$ we have:

$$x = \sum_{n=1}^{\infty} \frac{2(-1)^{n+1}}{n\pi} \sin(n\pi x)$$

$$|x| = \frac{1}{2} - \sum_{n=1}^{\infty} \frac{2(1-(-1)^n)}{n^2\pi^2} \cos(n\pi x)$$

3) Let $f_e(x)$ be the even extension of

$$f(x) = \sin\left(\frac{x}{2}\right) \qquad 0 \le x \le \pi$$

a) Find the Fourier series for $f_e(x)$. Does it converge pointwise? Uniformly? (Remeber that:

$$\int \sin(\lambda x)\cos(\mu x)dx = \frac{\cos(\mu-\lambda)x}{2(\mu-\lambda)} - \frac{\cos(\mu+\lambda)x}{2(\mu+\lambda)}$$

if $\mu \neq \lambda$)

b) Find the Fourier series for $f_e'(x)$. Does it converge pointwise? Uniformly?

c) Write an expression for $f_e''(x)$ and its Fourier series. (**Hint**: remeber the discontinuity at $x = 0$)

d) **(Bonus)** Can you use the relation between $f_e(x)$ and $f_e''(x)$ to compute the Foureir series of $f_e(x)$?