Math 1553 Worksheet §1.2, §1.3

1. Solve the following system of equations by putting an augmented matrix into reduced row echelon form. State which variables (if any) are free variables.

$$x_1 + 3x_2 + x_3 = 1$$
$$-4x_1 - 9x_2 + 2x_3 = -1$$
$$-3x_2 - 6x_3 = -3.$$

2. Consider the system of linear equations

$$\begin{aligned} x + 2y &= 7 \\ 2x + y &= -2 \\ -x - y &= 4 \end{aligned}$$

Question: Does this system have a solution? If so, what is the solution set?

   a) Formulate this question as an augmented matrix.

   b) Formulate this question as a vector equation.

   c) What does this mean in terms of spans?

   d) Answer the question using the interactive demo.

   e) Answer the question using row reduction.

3.

Zander has challenged you to find his hidden treasure, located at some point (a, b, c). He has honestly guaranteed you that the treasure can be found by starting at the origin and taking steps in directions given by

$$v_1 = \begin{pmatrix} 1 \\ -1 \\ -2 \end{pmatrix} \qquad v_2 = \begin{pmatrix} 5 \\ -4 \\ -7 \end{pmatrix} \qquad v_3 = \begin{pmatrix} -3 \\ 1 \\ 0 \end{pmatrix}.$$

By decoding Zander's message, you have discovered that the treasure's first and second entries are (in order) -4 and 3.

a) What is the treasure's full location?

b) Give instructions for how to find the treasure by only moving in the directions given by v_1, v_2, and v_3.