Math 1553 Worksheet §2.8, 2.9

1. Let $A = \begin{pmatrix} 1 & 3 & 2 \\ 2 & 6 & 4 \end{pmatrix}$ and let T_A be the corresponding linear transformation, defined by $T_A(v) = Av$.

a) Find a basis for the nullspace of A and roughly sketch Nul A.

b) Using the previous part, determine if T_A is one-to-one.

c) Find a basis for the column space of A and draw a picture of Col A.

d) Using the previous part, determine if T_A is onto.

e) Given that we know the nullspace and column space, we can describe the transformation T_A as follows:

The transformation T_A squashes $\mathbf{R}^{\square}$ *to a* ______ *by crushing parallel* ______ *to points. Its range is a* ______ *in* $\mathbf{R}^{\square}$.

2. Answer "YES" if the statement is always true, "NO" if it is always false, and "MAYBE" otherwise.

a) If A is a 3×100 matrix of rank 2, then $\dim(\mathrm{Nul} A) = 97$.

YES NO MAYBE

b) If A is an $m \times n$ matrix and $Ax = 0$ has only the trivial solution, then the columns of A form a basis for $\mathbf{R}^m$.

YES NO MAYBE

3. Consider the following vectors in $\mathbf{R}^3$:

$$b_1 = \begin{pmatrix} 2 \\ 2 \\ 2 \end{pmatrix} \qquad b_2 = \begin{pmatrix} 1 \\ 4 \\ 3 \end{pmatrix} \qquad u = \begin{pmatrix} 1 \\ 10 \\ 7 \end{pmatrix}$$

Let $V = \mathrm{Span}\{b_1, b_2\}$.

a) Explain why $\mathcal{B} = \{b_1, b_2\}$ is a basis for V.

b) Determine if u is in V. If so, find $[u]_{\mathcal{B}}$ (the $\mathcal{B}$-coordinates of u).