MATH 1553, SPRING 2018
SAMPLE MIDTERM 1: THROUGH SECTION 1.5

Name		**Section**	

Please **read all instructions** carefully before beginning.

- You have 50 minutes to complete this exam.
- There are no aids of any kind (calculators, notes, text, etc.) allowed.
- Please show your work unless specified otherwise. A correct answer without appropriate work may be given little or no credit.
- You may cite any theorem proved in class or in the sections we covered in the text.
- Good luck!

This is a practice exam. It is similar in format, length, and difficulty to the real exam. It is **not** meant as a comprehensive list of study problems. I recommend completing the practice exam in 50 minutes, without notes or distractions.

Problem 1.

[Parts a) through e) are worth 2 points each]

True or false. Circle **T** if the statement is **always** true, and circle **F** otherwise. You do not need to justify your answer.

a) **T** **F** If $Ax = b$ is consistent, then the equation $Ax = 5b$ is consistent.

b) **T** **F** If a system of linear equations has more variables than equations, then the system must have infinitely many solutions.

c) **T** **F** If A is an $m \times n$ matrix and $Ax = 0$ has a unique solution, then $Ax = b$ is consistent for every b in $\mathbf{R}^m$.

d) **T** **F** The three vectors $\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$, $\begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$, and $\begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix}$ span $\mathbf{R}^3$.

e) **T** **F** If A is a 5×3 matrix, then it is possible for $Ax = 0$ to be inconsistent.

Problem 2.

Parts (a), (b) and (c) are 2 points each. Parts (d) and (e) are 3 points each.

a) Compute $\begin{pmatrix} 3 & 2 \\ -2 & 0 \\ 1 & 4 \end{pmatrix}\begin{pmatrix} 1 \\ -3 \end{pmatrix}$

b) If A is a 2×3 matrix with 2 pivots, then the set of solutions to $Ax = 0$ is a:

(circle one answer) point line 2-plane 3-plane

in:

(circle one answer) $\mathbf{R}$ $\mathbf{R}^2$ $\mathbf{R}^3$.

c) Write a vector equation which represents an inconsistent system of two linear equations in x_1 and x_2.

d) Write a vector in $\mathbf{R}^3$ which is not a linear combination of $v_1 = \begin{pmatrix} 4 \\ 1 \\ 1 \end{pmatrix}$ and $v_2 = \begin{pmatrix} 5 \\ 2 \\ 2 \end{pmatrix}$.

Justify your answer.

e) If b, v, w are vectors in $\mathbf{R}^3$ and $\text{Span}\{b, v, w\} = \mathbf{R}^3$, is it possible that b is in $\text{Span}\{v, w\}$? Justify your answer.

Problem 3. [10 points]

Johnny Rico believes that the secret to the universe can be found in the system of two linear equations in x and y given by

$$x - y = h$$
$$3x + hy = 4$$

where h is a real number.

a) Find all values of h (if any) which make the system inconsistent. Briefly justify your answer.

b) Find all values of h (if any) which make the system have a unique solution. Briefly justify your answer.

Problem 4. [11 points]

a) Solve the system of equations by putting an augmented matrix into reduced row echelon form. Clearly indicate which variables (if any) are free variables.

$$x_1 + 2x_2 + 2x_3 - x_4 = 4$$
$$2x_1 + 4x_2 + x_3 - 2x_4 = -1$$
$$-x_1 - 2x_2 - x_3 + x_4 = -1$$

b) Write the set of solutions to

$$x_1 + 2x_2 + 2x_3 - x_4 = 0$$
$$2x_1 + 4x_2 + x_3 - 2x_4 = 0$$
$$-x_1 - 2x_2 - x_3 + x_4 = 0$$

in parametric vector form.

Problem 5.

[7 points]

Write an augmented matrix corresponding to a system of two linear equations in three variables x_1, x_2, x_3, whose solution set is the span of $\begin{pmatrix} -4 \\ 1 \\ 0 \end{pmatrix}$.

Briefly justify your answer.

[Scratch work]