Name:____________________________ Recitation Section:__________

Math 1553 Lecture C, Quiz 4: Linear transformations (10 points, 10 minutes)

Solutions

Show your work and justify answers where appropriate. If you write the correct answer without sufficient work or justification, you will receive little or no credit.

1. (1 point each) True or False. Circle TRUE if the statement is *always* true. Otherwise, circle FALSE. You do not need to justify your answer.

a) If T is the linear transformation whose standard matrix is $\begin{pmatrix} 1 & 0 & 2 \\ -3 & 1 & 5 \end{pmatrix}$, then the domain of T is $\mathbf{R}^2$.

TRUE $\boxed{\text{FALSE}}$

b) If $T : \mathbf{R}^3 \to \mathbf{R}^2$ is a linear transformation, then T is onto.

TRUE $\boxed{\text{FALSE}}$ (for example, $T(x,y,z) = (x,0)$)

2. (3 points) Write the standard matrix A for the transformation $T : \mathbf{R}^2 \to \mathbf{R}^2$ that first rotates counterclockwise by $\frac{\pi}{2}$ radians, then reflects about the line $y = x$.

Show your work!

Solution.

$A = \begin{pmatrix} T\begin{pmatrix}1\\0\end{pmatrix} & T\begin{pmatrix}0\\1\end{pmatrix} \end{pmatrix}.$

The rotation sends $\begin{pmatrix}1\\0\end{pmatrix}$ to $\begin{pmatrix}0\\1\end{pmatrix}$, then reflection about $y = x$ sends $\begin{pmatrix}0\\1\end{pmatrix}$ to $\begin{pmatrix}1\\0\end{pmatrix}$, so $T\begin{pmatrix}1\\0\end{pmatrix} = \begin{pmatrix}1\\0\end{pmatrix}$.

The rotation sends $\begin{pmatrix}0\\1\end{pmatrix}$ to $\begin{pmatrix}-1\\0\end{pmatrix}$, then reflection about $y = x$ sends $\begin{pmatrix}-1\\0\end{pmatrix}$ to $\begin{pmatrix}0\\-1\end{pmatrix}$, so $T\begin{pmatrix}0\\1\end{pmatrix} = \begin{pmatrix}0\\-1\end{pmatrix}$.

Thus $A = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$.

Alternatively, we recently learned matrix multiplication. We put the rotation matrix on the right and the reflection matrix on the left:

$$A = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}.$$

(Turn over to the back side for problem 3!)

3. (5 points) Let $T : \mathbf{R}^3 \to \mathbf{R}^3$ be the linear transformation $T\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 3x \\ y+z \\ -2x \end{pmatrix}$.

a) Write the standard matrix A for T.

b) Is T one-to-one? If your answer is yes, justify why. If your answer is no, find two different vectors v and w so that $T(v) = T(w)$.

Solution.

a) $A = \begin{pmatrix} T(e_1) & T(e_2) & T(e_3) \end{pmatrix} = \begin{pmatrix} 3 & 0 & 0 \\ 0 & 1 & 1 \\ -2 & 0 & 0 \end{pmatrix}$

b) No. The matrix A only has two pivots, so it doesn't have a pivot in every column and thus T isn't one-to-one. We could skip the justification and give a counterexample to show that T cannot be one-to-one.

If v and w are vectors in $\mathbf{R}^3$, then $T(v) = T(w)$ if and only if the first entries of v and w are equal and their second and third entries sum to the same number. For example, $T\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} = T\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$, while $T\begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix} = T\begin{pmatrix} 0 \\ 2 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 2 \\ 0 \end{pmatrix}$, etc.