Shalen

– Joint with Rosemary Guzman

* Rk of G is min size of gen set.

* Γ is $k(\in \mathbb{N})$-free if every subg of rk atmost k is free.

* Γ is not free but k-free $\Rightarrow rk(\Gamma) > k$.

* Thm: (Culler-S, Agol). $k \geq 3$, M^3 closed, orientable & hyp. $\dim(H_1(M;\mathbb{Z}_2)) \geq \max(3k-4, 6)$. Then either $\pi_1(M)$ is k-free or M contains a closed incompressable surface of some genus $1 < g < k$.

* $S \underset{\text{Finite}}{\subset} \Gamma$.

  Defn: ~~Infinite~~ Internal rank of S is $\max_{T \subset S} rk(\langle T \rangle)$

* $p \in M$ hyp 3-mfd. $\lambda > 0$. Defn $S_p(\lambda) \subset \pi_1(M^3, p)$ the set of all indivisible elt that have +ve powers repn by loops of length $< \lambda$.

Quantitative Mostow-Rigidity

* Thm (G-S): Suppose M^3 is closed, orientable, 3-mfd. $\pi_1(M)$ is k-free. Then $\exists$ a pt $p \in M$ s.t. the internal rank $S_p(\log(2k-1))$ is atmost $k-3$.

* $k=3$. $\pi_1(M)$ 3-free $\Rightarrow$ ~~$S(M)=\phi$~~ $S_p(\log 5) = \phi$ for some p.

  i.e $M \supset$ hyp ball of radius $(\log 5)/2$.

Proved by Anderson, Canary, Culler, Shalen.

mod Mardea Conj. (Agol, Calegari-Gabai).

* This implies $Vol(M) > 3.08$.

This can be used to show if $Vol(M) \leq 3.08$ then $\dim(M;\mathbb{Z}_2) \leq 5$.

* $K=4$, $\exists P$ s.t $Sp(\log(7)) \subset$ cyclic gp.
* Culler - S used to prove 4-free $\Rightarrow$ Vol > 3.44.
* Corollary: If $Vol(M) < 3.44$ then $\dim H_1(M;\mathbb{Z}_2) \leq 7$.
* Agol, Leininger, Margalit: M^3 closed, hyp $\Rightarrow \dim(H_1(M,\mathbb{Z}_p)) < 334.08\, Vol(M)$.

* New thm improves Guzmann's result for $K<5$.
  Should play a role in getting lower bdd for $\dim H_1(M;\mathbb{Z}_2)$ in terms of $vol(M)$.
* Log(2K-1) thm (ACCS, mod Marden conj.)
* Let $x_1, \dots, x_K$ be elts of o.p isom gp. of $\mathbb{H}^3$ s.t $\langle x_1, \dots, x_K\rangle$ is discrete & free of rk. K.
  (And purely laxodromic) Then for every pt $z \in \mathbb{H}^3$, $d_i = dist(z, x_i z)$. $\sum_{i=1}^{K} \frac{1}{1+e^{d_i}} \leq 1/2$. In particular, for i,
  $d_i \geq \log(2K-1)$.

Pf. of new thm: C max. cyclic subgp. of Γ. $M = \mathbb{H}^3/\Gamma$ Γ – discrete + torsion free.

$Z_C(\lambda) = \{z \mid d(z, \lambda z) < c\}$

If conclusion is false, $Z_C(\lambda)$ cover $\mathbb{H}^3$.
Study nerve of the covering. K Borsuk-nerve theorem
$\Rightarrow$ Contractible.

Internal rank of simplex σ = Int rank of set of max. cycl. to vertices.

log(2K-1) thm $\Rightarrow$ Int rank of any simplex $<K$.

L = union of all simplices of int rk $\leq K-3$
subcomplex.

. If the conclusion is false, $\Rightarrow$ the link in K of every simplex is contractible.

$\therefore |K|-|L| \Rightarrow$ contractible.

Union of simplices of dim $K-1$ & $K-2$.

X_r = union of simplices of internal rk, $r = K-1, K-2$

Action of Γ on a tree (w/o inversion)

Combinatorial argument $\Rightarrow$ Stab. are locally free.

($\Rightarrow \Leftarrow$)

"Log(2k-1) - Theorem Paper"

Anderson, Canary, Culler, Shalen

Agol-Culler-Shalen

Vol & Mod-p homology