## Introduction

**Nielsen-Thurston:** Every braid has a number associated to it called its entropy, which is a characterization of how complicated the braid is.

## The Problem

### Results

\[
L(BI_n) \preceq 1 \\
L(B_n[m]) \preceq 1 \\
L([B_n, B_n]) \to 0 \quad \text{as} \quad n \to \infty
\]

## The Braid Torelli Group

\[
BI_n = \ker(B_n \to Sp_{2n}(\mathbb{Z}))
\]

## Level-m Congruence Subgroups of the Braid Group

\[
BI_n \preceq B_n[m] \preceq \text{Mod}(S^b_g[m])
\]

## Methods

Braids ♦ Train Tracks ♦ Matrices

Converting complicated curve to train track.

The corresponding train track for the braid in the commutator subgroup used to show our third result.

## Acknowledgements

We would like to thank the Georgia Tech Mathematics REU for this opportunity, and Dr. Dan Margalit and Dr. Hyunshik Shin for their mentorship. This research is supported by the NSF.