# Lightning Talks I
# Tech Topology Conference

December 8, 2017

# Braids with Boundary-Parallel Strands

Michael Dougherty (UCSB)

December 8, 2017

Tech Topology Conference

Brady 2001, Brady-McCammond 2010:

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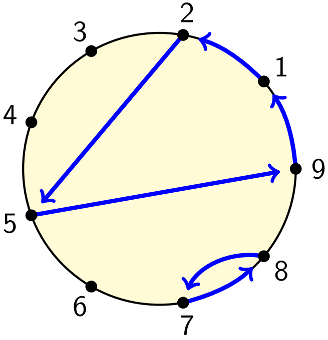
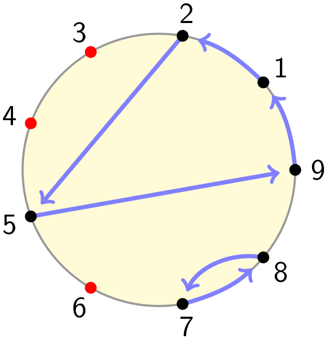
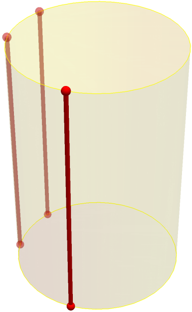
**Idea:** Understand the curvature of large subcomplexes

$\text{BRAID}_n$ is the fundamental group of a configuration space



# Fixed strands



strand is fixed $\leftrightarrow$ $\exists$ a braid isotopy to the constant path

**Thm:** (D-McCammond-Witzel)

Fixed strands can be fixed simultaneously.

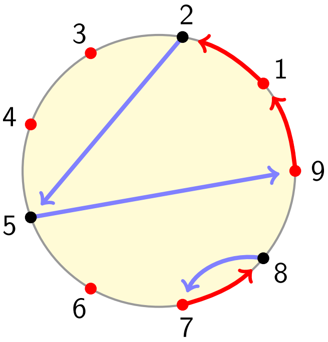
Choose $k$ strands to fix $\rightsquigarrow$ subgroup of $\textsc{Braid}_n$

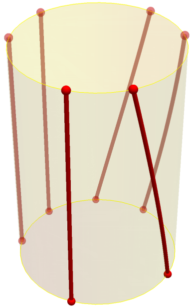
$\rightsquigarrow$ subcomplex of $\textsc{Cplx}(\textsc{Braid}_n)$

**Thm:** (Brady-McCammond 2010)

The corresponding subcomplex is $\textsc{Cplx}(\textsc{Braid}_{n-k})$.

# Generalization: Strands which stay in the boundary



strand is boundary-parallel $\leftrightarrow$ $\exists$ a braid isotopy into the boundary

**Thm:** (D-McCammond-Witzel)

Boundary-parallel strands are simultaneously boundary-parallel.

$$\begin{aligned} \text{Choose } k \text{ strands} \quad &\rightsquigarrow \quad \text{subset of } \textsc{Braid}_n \\ &\rightsquigarrow \quad \text{subcomplex of } \textsc{Cplx}(\textsc{Braid}_n) \end{aligned}$$

**Thm:** (D-McCammond-Witzel)

The corresponding subcomplex is $\textsc{Cplx}(\textsc{Braid}_{n-k}) \times \Delta^{k-1} \times \mathbb{R}$.

# Lightning Talks I
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# Link Homologies of Infinite Braids

Michael Abel, Duke University
Michael Willis, UCLA

December 8, 2017

# The Infinite Twist

For many link homology theories (Khovanov, Khovanov-Rozansky, HOMFLY, certain 'colored' versions), the *infinite twist* on $n$ strands is frequently studied.

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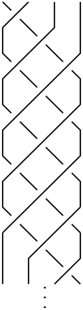

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1. Jones-Wenzl projector $P_n \Rightarrow$ quantum invariants of 3-mflds
2. Jones polynomial of infinite twist 'builds' $P_n$
3. Categorify it (Khovanov $\mathfrak{sl}_2$)
4. Allow other Lie groups (Khovanov-Rozansky $\mathfrak{sl}_n$) $\Rightarrow$ categorify highest weight projectors from representation theory (Rose, Cautis, Hogancamp)

# Why?

When we close large twists in the usual way, we get torus links. Then the infinite twist looks at the stable link homologies of torus links as the twisting goes to infinity.

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When we close large twists in the usual way, we get torus links. Then the infinite twist looks at the stable link homologies of torus links as the twisting goes to infinity.

Link homologies of torus links have also been studied and computed extensively (Stošić, Elias, Hogancamp, Mellit) and conjecturally relate to several other fields of mathematics (Gorsky, Oblomkov, Rasmussen, Shende, etc).

# The Question

What about *other* positive infinite braids?

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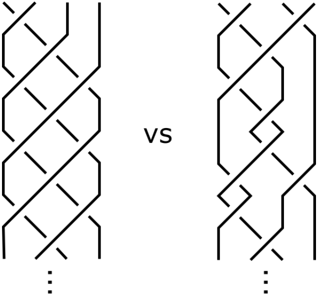

# The Answer

**Theorem (Abel,W.)**

*For any of the $\mathfrak{sl}_n$ and HOMFLY link homologies (including certain 'colored' versions), any complete, positive, infinite braid $\mathcal{B}_\infty$ on $n$ strands gives the same stable limiting homology groups as the infinite twist $H^*(\mathcal{B}_\infty) \cong H^*(\mathcal{T}^\infty)$.*

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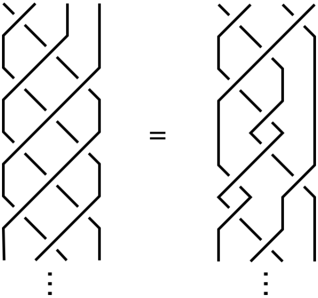

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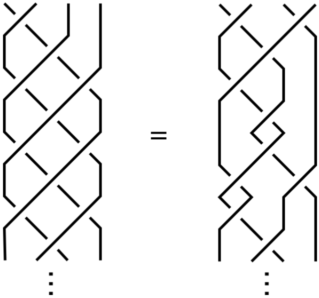


Here, *complete* means every positive braid group generator appears infinitely often (no crossings stop appearing).

# Overview of Proof

Idea is to compare larger and larger finite sub-braid $\mathcal{B}_\ell$ to finite twists $\mathcal{T}^k$ for larger and larger $k$.

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- $C^*(\mathcal{B}_\ell) = \big(C^*(\mathcal{T}^k) \to C^*(\mathrm{Errors})\big)$
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- Show that the $C^*(\mathrm{Errors})$ terms get pushed out 'further and further to the right' (larger and larger homological degree) as $\ell$ and $k$ grow
- Limit as $\ell$ and $k$ go to infinity together

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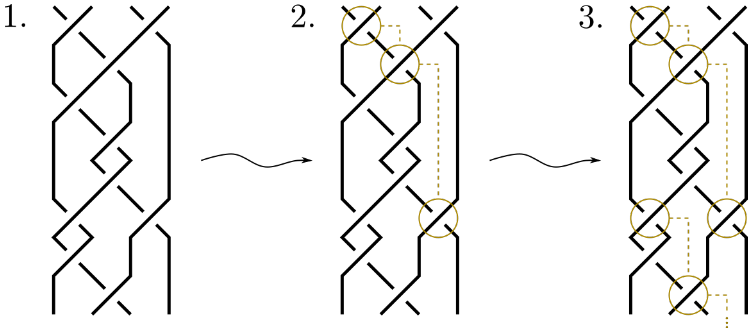

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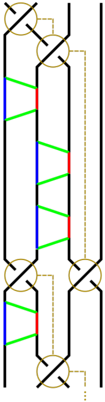

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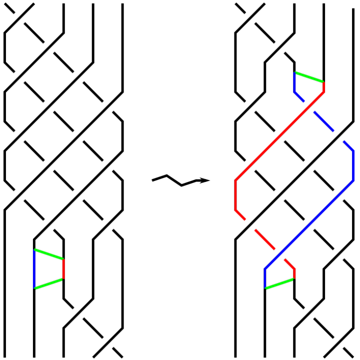

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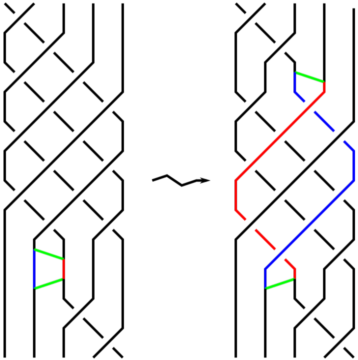

Homological shifts due to different colors then push these 'error' diagrams into larger and larger homological degrees. Done!

# End!

Thank you!!

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# Obstructions to Riemannian smoothings of locally CAT(0) 4-manifolds

Bakul Sathaye

Ohio State University

Tech Topology Conference

Dec 8, 2017

Closed manifolds with non-positive curvature:

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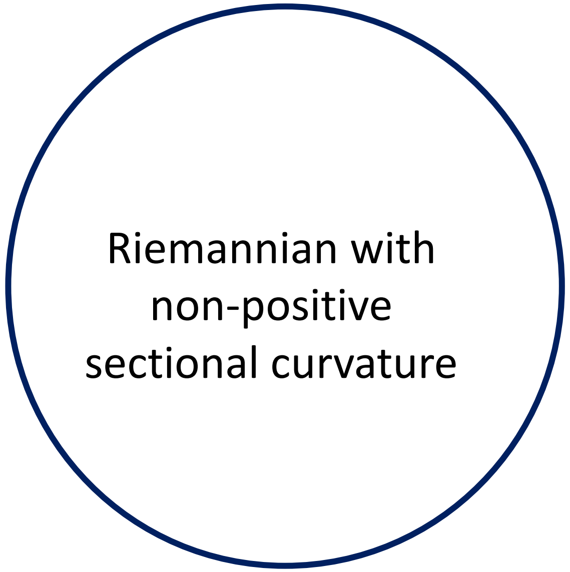
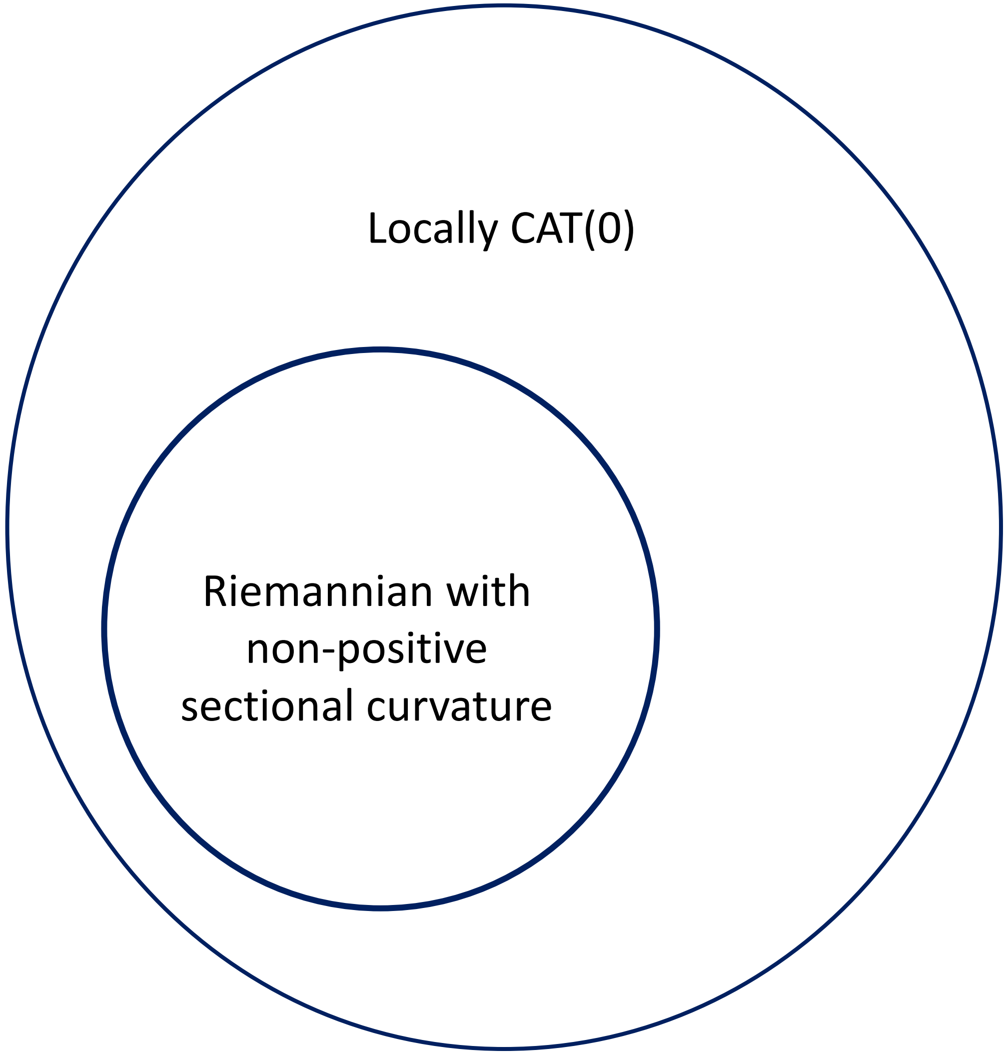
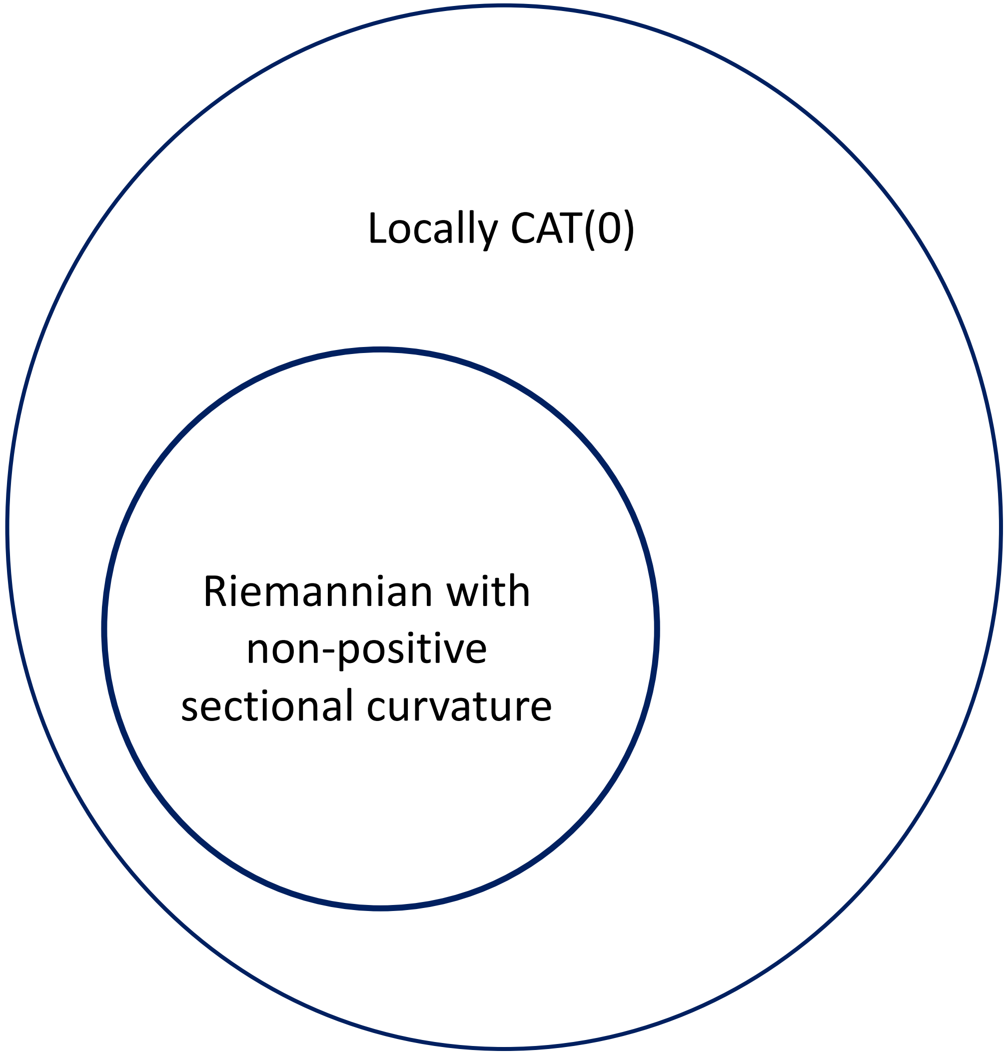


# Closed manifolds with non-positive curvature:



Closed manifolds with non-positive curvature:

Dim = 4



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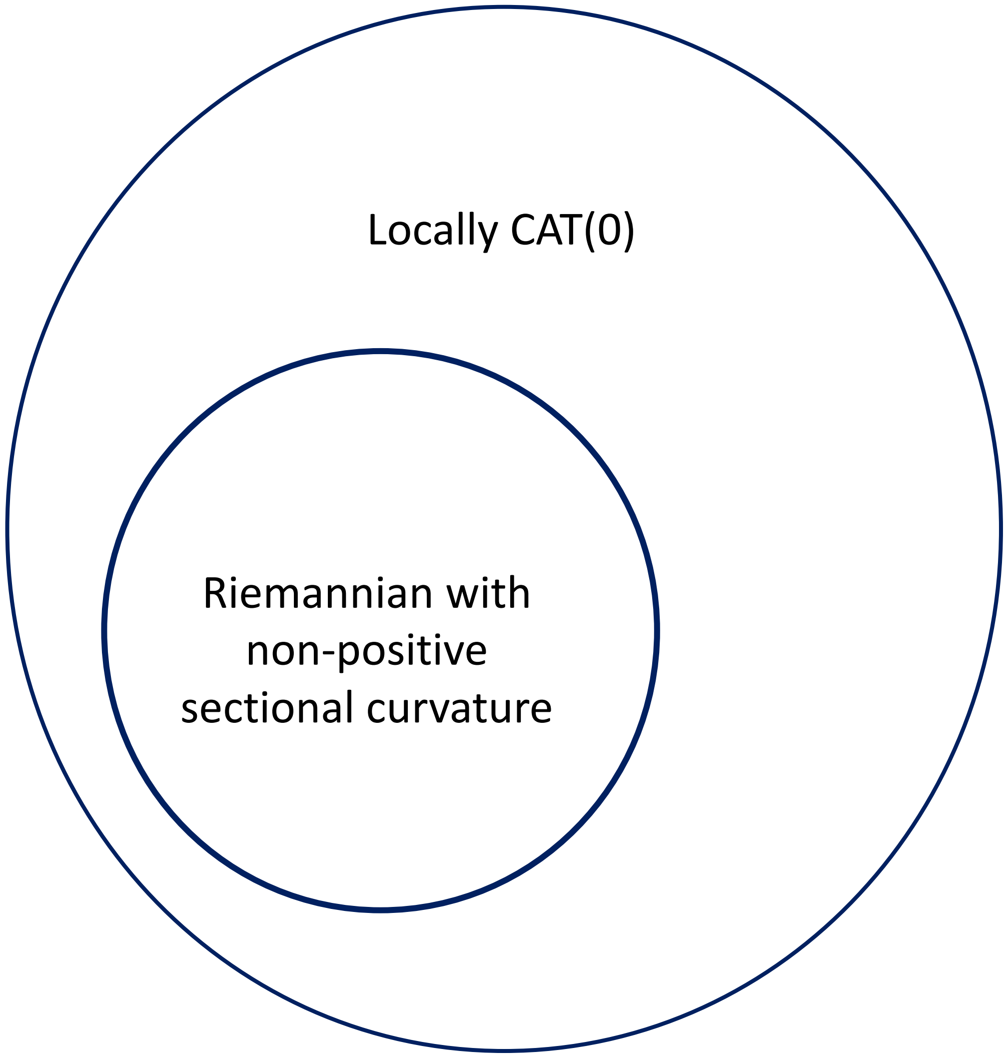


## Dim = 4

(Davis – Januszkiewicz – Lafont)

If M is a locally CAT(0) manifold with isolated flats,

$\exists$ a flat $F \subset \widetilde{M}$ such that $\partial^\infty F$ is a non-trivial knot in $\partial^\infty \widetilde{M} \cong \mathbb{S}^3$

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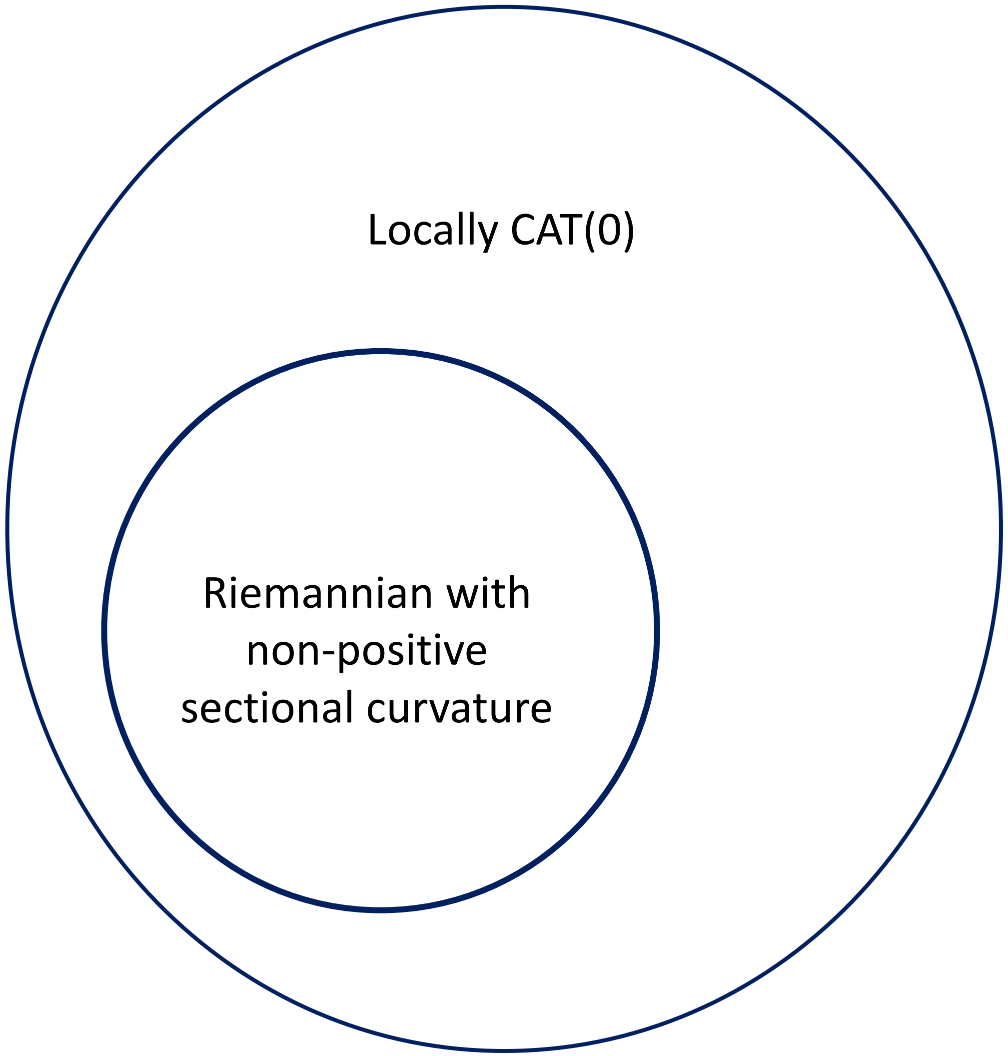


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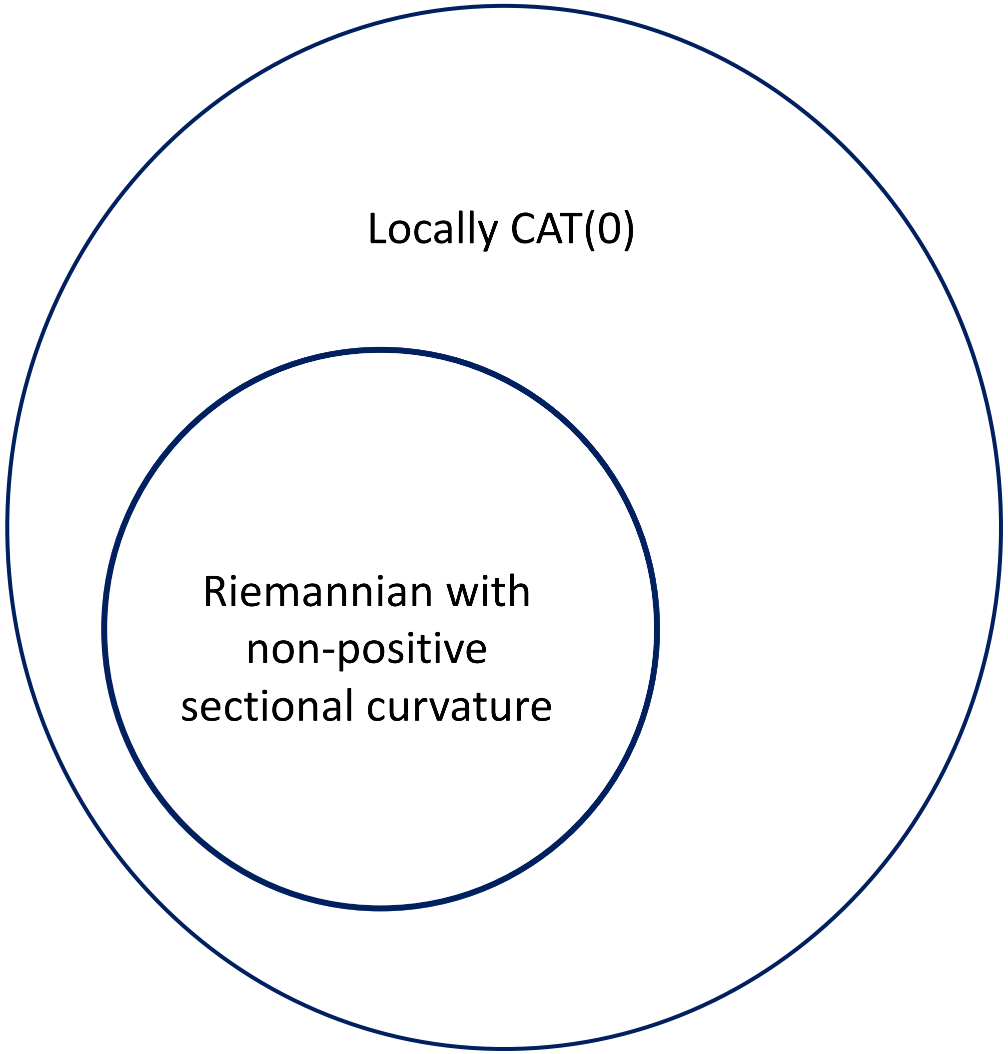


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$\exists$ a collection of flats $F_1, \dots, F_n \subset \widetilde{M}$ such that
$\partial^{\infty} F_1, \dots, \partial^{\infty} F_n$ form a non-trivial link $L$ in $\partial^{\infty} \widetilde{M} \cong \mathbb{S}^3$ that is not a great circle link.

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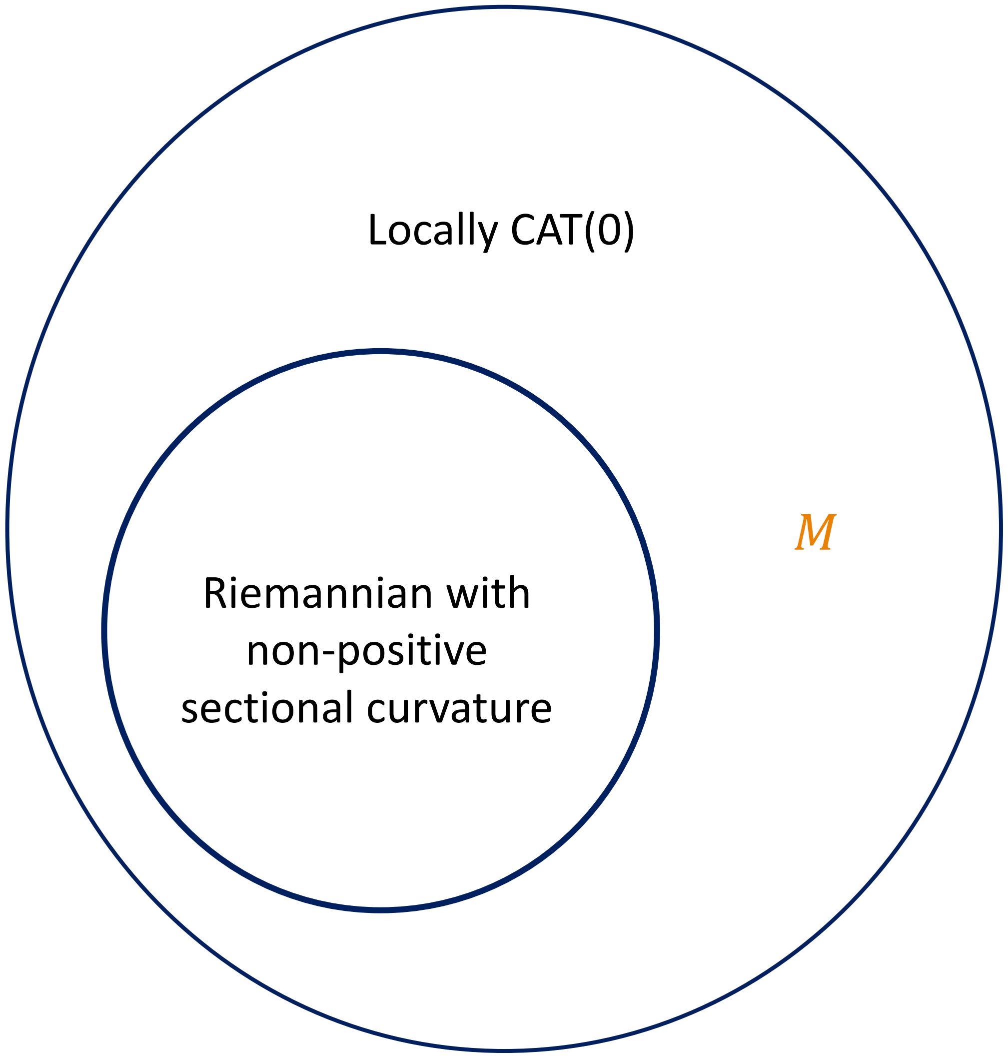


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→ has $n$ isolated squares- one for each $l_i$

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$\widetilde{\boldsymbol{M}}$ = Davis complex for $\Sigma$

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$\boldsymbol{P_\Sigma} = \boldsymbol{M}$ → Locally CAT(0) manifold

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$\widetilde{\boldsymbol{M}}$ → *1.* $\widetilde{M} \cong \mathbb{R}^4$ and $\partial^\infty \widetilde{M} \cong \mathbb{S}^3$

(Davis complex)

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$\widetilde{\boldsymbol{M}}$ (Davis complex) ⟶

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2. Each vertex $v$ of $\widetilde{M}$ contains a copy of link $L$ in its link (local)
3. $\widetilde{M}$ contains $n$ flats $F_1, \dots, F_n$ so that their boundaries form the link $L$ (S.) (global)

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3. $\widetilde{M}$ contains $n$ flats $F_1, \dots, F_n$ so that their boundaries form the link $L$ (S.) — global
4. $\widetilde{M}$ has the Isolated Flats property (Caprace)

If $M$ homotopy equivalent to
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(Hruska-Kleiner – using Isolated flats)

(flat torus theorem)

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For $p \in \widetilde{M}$, geodesic retraction

$$\mathbb{S}^3 \cong \partial^\infty \widetilde{M'} \xrightarrow{homeo} T_p\widetilde{M'} \cong \mathbb{S}^3$$ (unit tangent space)

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local

For $p \in \widetilde{M}$, geodesic retraction

$$\mathbb{S}^3 \cong \partial^\infty \widetilde{M'} \xrightarrow{homeo} T_p \widetilde{M'} \cong \mathbb{S}^3 \text{ (unit tangent space)}$$

$$L \longmapsto T_p F_1', \ldots, T_p F_n'$$

(great circle link)

$L \neq$ great circle link

$\rightarrow\leftarrow$

If $M$ homotopy equivalent to
$M'$ = Riemannian manifold, $\kappa \leq 0$

$\exists$ flats $F_1', \dots, F_n' \subset \widetilde{M}'$ such that $\partial^\infty F_1', \dots, \partial^\infty F_n'$ is isotopic to the link $L$

(Hruska-Kleiner – using Isolated flats)

(flat torus theorem)

global

But!

local

For $p \in \widetilde{M}$, geodesic retraction

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$$L \neq \text{great circle link}$$

$$\rightarrow\leftarrow$$

**Thank you!**

# Lightning Talks I
# Tech Topology Conference

December 8, 2017

# On the Chebyshev homomorphism

## and topological quantum compiling

Jonathan Paprocki

December 6, 2017

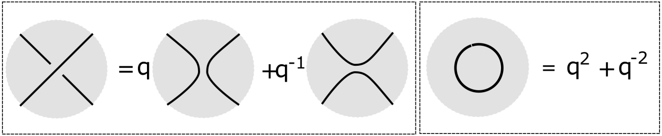
1. Background
   - Skein algebras
   - Chebyshev homomorphism
   - Center of skein algebra

2. Compiling
   - Topological quantum compiling
   - Relation to center

Let $\Sigma$ be a surface with boundary and punctures, and let $R = \mathbb{C}[q^{\pm 1/2}]$.

**Definition (Przytycki, Turaev)**

*The* **Kauffman bracket skein module** $\mathcal{S}_q(\Sigma)$ *of* $\Sigma$ *is the* $R$*-module freely spanned by homotopy classes of framed links in* $\Sigma \times (-1, 1)$ *(including* $\emptyset$*), modulo the usual* **skein relation** *and* **trivial loop relation***.*




$\mathcal{S}_q(\Sigma)$ is also an $R$-algebra. $\alpha \cdot \beta$ is defined to be the link $\alpha \cup \beta$ obtained by rescaling $\alpha$ to lie in $(-1, 0)$ and $\beta$ to lie in $(0, 1)$. We will consider $q$ to be a root of unity.

### Theorem (Bonahon, Wong)

*Suppose* $\xi^4$ *is a primitive* $N$*th root of unity. Let* $\epsilon = \xi^{N^2}$. *Then there is a unique* $\mathbb{C}$*-algebra homomorphism* $\mathbf{Ch} : \mathcal{S}_\epsilon(\Sigma) \to \mathcal{S}_\xi(\Sigma)$ *such that for any framed link* $L = K_1 \cup \cdots \cup K_m \subset M$,

$$\mathbf{Ch}(L) = T_N(K_1) \cup \cdots \cup T_N(K_m).$$

$\mathbf{Ch}$ *is called the* **Chebyshev homomorphism**.

### Remark

$\epsilon$ *is a 4th root of unity.* $T_N$ *is the* $N$*th Chebyshev polynomial of the first type.*

**Theorem (Frohman, Kania-Bartoszynska, Lê)**

*Let $\partial$ be the set of knots surrounding a puncture, and $n$ the order of $\xi$. If $n \neq 0$ mod 4, the center $Z_\xi(\Sigma)$ of $\mathcal{S}_\xi(\Sigma)$ is generated by the image of* $\mathbf{Ch} : \mathcal{S}_\epsilon(\Sigma) \to \mathcal{S}_\xi(\Sigma)$ *and $\partial$.*

**Remark**

*If $n = 0$ mod 4, $Z_\xi(\Sigma)$ is generated by* **Ch** *of a particular subalgebra of $\mathcal{S}_\epsilon(\Sigma)$ and $\partial$.*

**Theorem (Frohman, Kania-Bartoszynska, Lê)**

*$\mathcal{S}_\xi(\Sigma)$ is finitely generated as a module over its center.*

**Remark**

*The representation theory of algebras which are finitely generated as a module over their center is well understood.*

- A quantum program is a (unitary) matrix $U$.

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- A quantum program is a (unitary) matrix $U$.
- A topological quantum computer essentially implements a quantum representation $\rho$ of the mapping class group of $\Sigma$.
- Topological quantum compiling is finding a "minimum complexity" mapping class group element $x$ such that $\|\rho(x) - U\| < \epsilon$.
- Mapping class groups act on skein algebras. Skein algebra representations thus induce mapping class group representations. We wish to study mapping class group representations via skein algebra representations.

Let $\{\alpha_i\}$ be skeins with integer coefficients that generate $\mathcal{S}_\xi(\Sigma)$ over $Z_\xi(\Sigma)$, $\rho : \mathcal{S}_\xi(\Sigma) \to \mathrm{Aut}(V)$, $U \in \mathrm{Aut}(V)$. Write $U$ as

$$U = \sum_i z_i \rho(\alpha_i), \quad z_i \in \mathbb{C}$$

Use the characterization of $Z_\xi(\Sigma)$ as the image of **Ch** to find minimimum complexity central elements $c_i$ with integer coefficients such that $\rho(c_i) \approx z_i$. This essentially involves studying how the Chebyshev polynomials act on cyclotomic fields. Use this information to “compile” in the induced quantum mapping class group representation.

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