Fibrations of 3-manifolds, tilings and nowhere continuous functions

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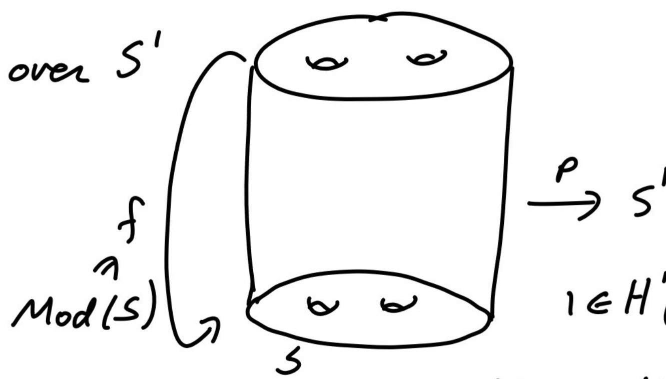
I. Calculus

exercise: Give an example for a function $f: \mathbb{Q} \to \mathbb{R}$ s.t.

- f is discontinuous at every $x \in \mathbb{Q}$
- $\bar{f}(x) = \lim_{x' \to x} f(x')$ exists for all $x \in \mathbb{R}$ and $\bar{f}$ is continuous

II 3-manifolds

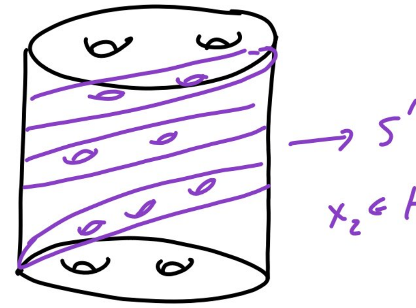
let M be a 3-manifold fibering over S^1



$1 \in H^1(S^1; \mathbb{Z})$

$x = p^*(1) \in H^1(M; \mathbb{Z})$

get different fibration



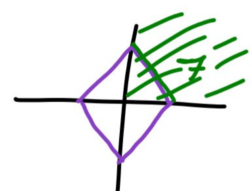
$x_2 \in H^1(M; \mathbb{Z})$

Thurston norm (70's, '86):

$\|\cdot\|_T$ on $H^1(M; \mathbb{R})$

semi-norm but norm if M hyperbolic

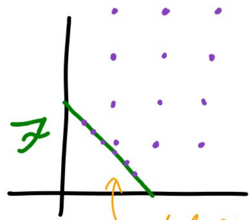
unit ball B is a finite sided polyhedron



if $x \in H^1(M)$ is fibered, then

$x \in \text{interior}(\mathbb{R}_+ F)$ for some top-dim'l face of B

and any other primitive integer point in $\text{int}(\mathbb{R}_+ F)$ corresponds to a fibration



radial proj $\mathbb{Z}$ latice to face
rational pts on $\mathcal{F}$

let g be a function from conjugacy classes of mapping classes to $\mathbb{R}$

get a function from rational points on $\mathcal{F}$ to $\mathbb{R}$ (pt $\mapsto$ fibration $\mapsto$ monodromy $\overset{g}{\mapsto} \mathbb{R}$)

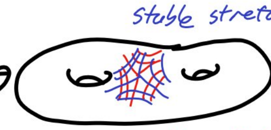
III Pseudo-Anosov mapping classes

Nielsen-Thurston classification th$^{\underline{m}}$

every $f \in \mathrm{Mod}(S)$ is either

1) finite order
2) pseudo-Anosov
3) reducible

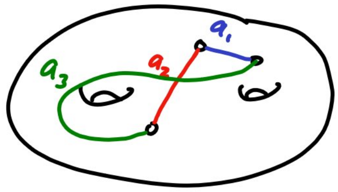
stable stretch $\frac{1}{\lambda}$

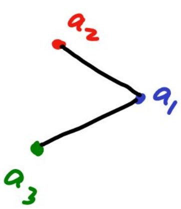


unstable stretch λ

Arc complex of a punctured surface

$\mathcal{A}(S)$

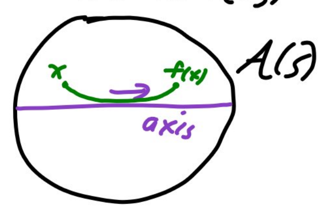




$\mathcal{A}(S)$ is a hyperbolic space and pseudo Anosov's act by translation along an axis in $\mathcal{A}(S)$

Asymptotic translation length of $f \in \mathrm{Mod}(S)$

$$\mathrm{ATL}(f) = \lim_{n\to\infty} \frac{d_{\mathcal{A}(S)}(x, f^n(x))}{x}$$



Th$^{\underline{m}}$ $\mathrm{ATL}(f) > 0 \iff f$ is pseudo-Anosov

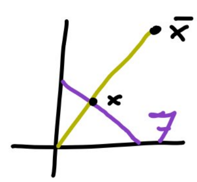
Th$^{\underline{m}}$ (Fried)

let M be a hyperbolic 3-manifold with $b_1(M) \geq 2$ and fibered face $\mathcal{F}$

the normalized stretch factor function

$$\mu_1 : \mathcal{F}_{\mathbb{Q}} \to \mathbb{R}_+$$

$$\mu_1(x) = \|\bar{x}\|_T \log \lambda(\bar{x})$$



extends to a continuous, convex function

$$\mathrm{int}(\mathcal{F}) \to \mathbb{R}_+$$

that goes to ∞ at ∂F

$$\Rightarrow \log \lambda_{\min,g} \leq \frac{C}{g} \qquad \text{(Penner '91)}$$

Th$^{\underline{m}}$ (S.):

let M be a fully-punctured hyp. 3-manifold

F a fibered face

Define the normalized ATL function as

$$\mu_2 : \mathcal{F}_{\mathbb{Q}} \longrightarrow \mathbb{R}_+$$
$$x \longmapsto \|x\|_T^2 \cdot ATL(\bar{x})$$

$\dim(\mathcal{F}) = 1$

suppose also $b_1(M) = 2$

① $\lim_{x' \to x} \mu_2(x')$ exists $\forall x \in \mathcal{F}$

and $\bar{\mu}_2(x) = \lim_{x' \to x} \mu_2(x')$ is convex

and continuous that goes to ∞ at ∂F

If a, b are endpoints of $\mathcal{F}$, then

$$\lim_{t' \to t} \mu_2\left(\frac{a+b}{2} + t\,\frac{b-a}{2}\right) = \frac{c}{(1+t)(1-t)}$$

for some $c > 0$

② $\exists M, \mathcal{F}$ s.t. $\bar{\mu}_2(x) \neq \mu_2(x)$ at ∞ many points (conjecture: all rational points)

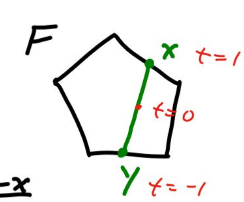
Th$^{\underline{m}}$ in progress: suppose $b_1(M) > 2$, $\dim(\mathcal{F}) > 1$

let $x, y \in \partial F_{\mathbb{Q}}$ s.t. $[x,y] \subset \mathrm{int}(\mathcal{F})$

and consider

$$\nu(t) = \frac{x+y}{2} + t\,\frac{y-x}{2}$$

defined on $(-1,1) \cap \mathbb{Q}$



(3) $\tilde{\nu}(t) = \lim_{t' \to t} \nu(t')$ exists for all $t \in (-1, 1)$

and there is $c(x,y) > 0$ s.t.

$$\tilde{\nu}(t) = \frac{c}{(1-t)(1+t)}$$

(4) $\lim_{x' \to x} \mu_2(x')$ does not exist anywhere.