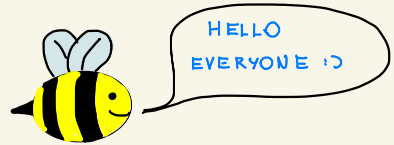
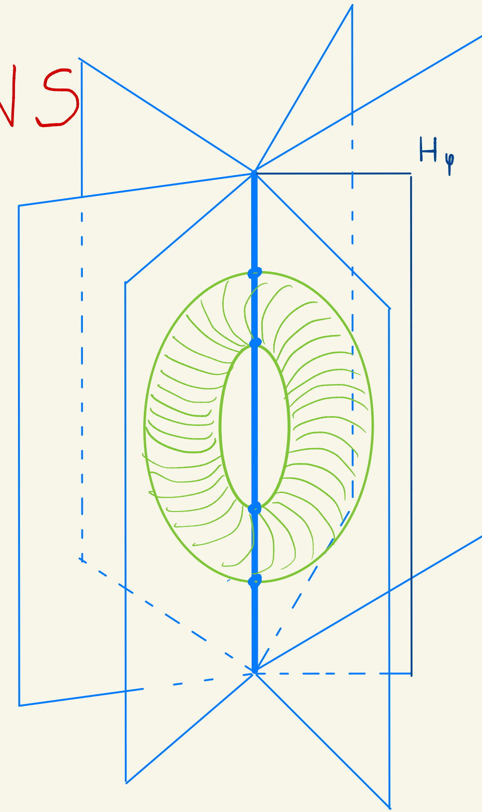
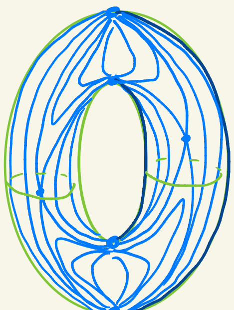
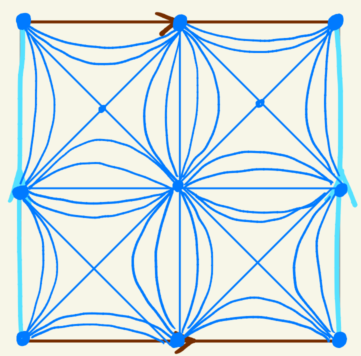
Recognising Open Book Foliations

Vera Vértesi
(University of Vienna)









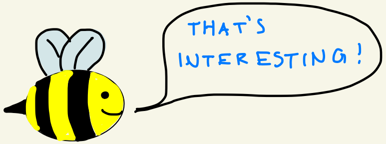
OVERVIEW

- open book foliation "=" open book $\cap$ surface

  oriented singular foliation

  Question: Which oriented singular foliations are open book foliations?

PLAN:



→ braid foliations

→ open book foliations

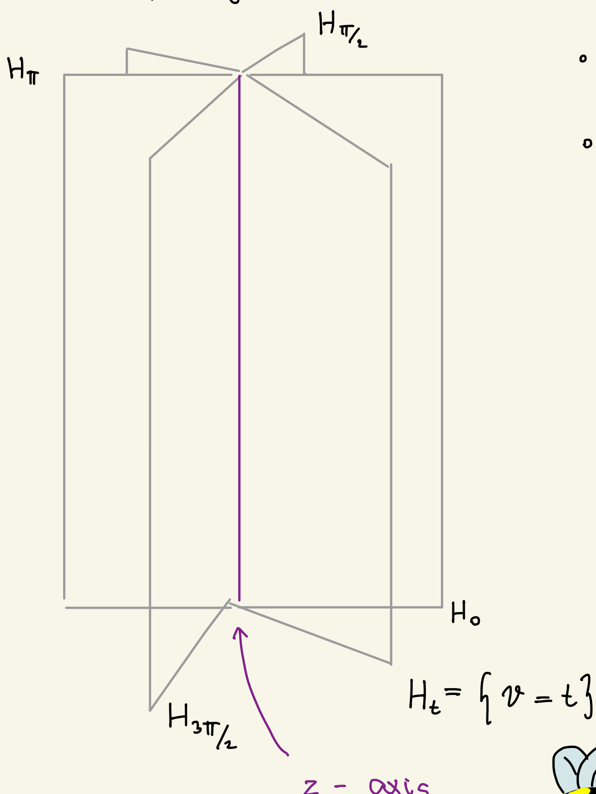
→ algorithm

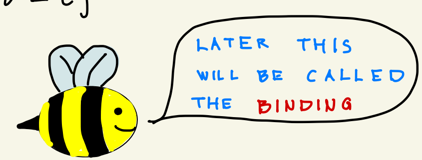
BRAID FOLIATIONS

Bennequin, Birman - Menasco, LaFountain

$\mathbb{R}^3$ w/ cylindrical coordinates $(z, r, \vartheta) \in \mathbb{R} \times \mathbb{R}_{\geq 0} \times S^1/_\sim$

- $H_t := \{\vartheta = t\}$ half planes
- they intersect in the z-axis





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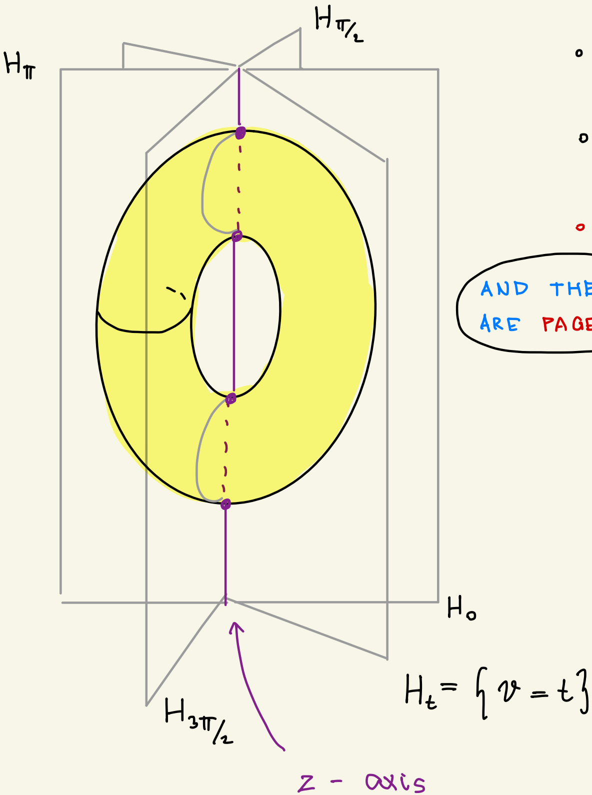

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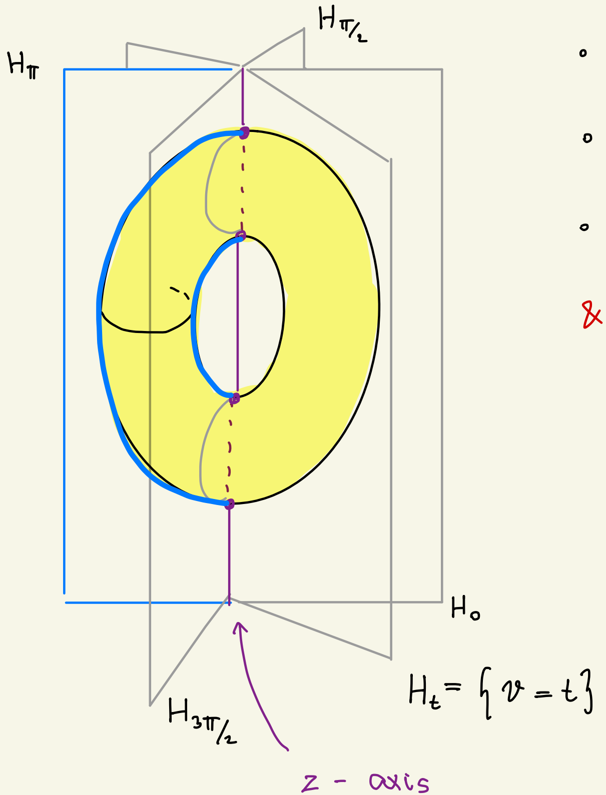

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& look at its intersection w/ H_t $t = \pi$

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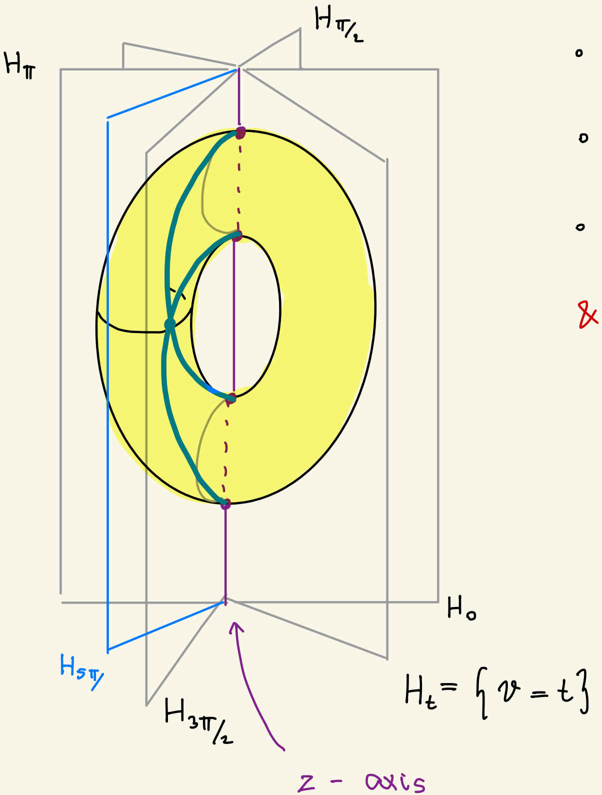

- $H_t := \{\vartheta = t\}$ half planes
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& look at its intersection w/ H_t $t = 5\pi/4$

BRAID FOLIATIONS

Bennequin, Birman · Menasco, LaFountain

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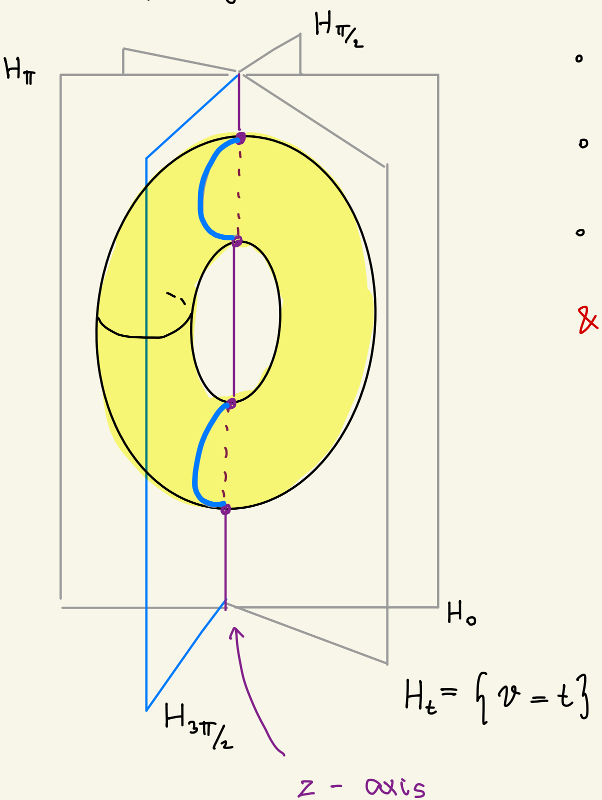

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BRAID FOLIATIONS

Bennequin, Birman - Menasco, LaFountain

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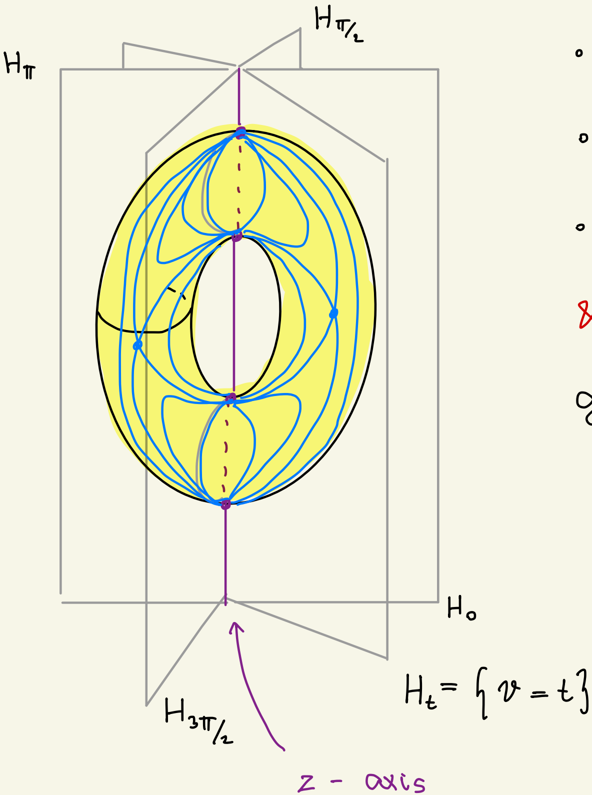

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gives a <u>singular foliation</u> on Σ

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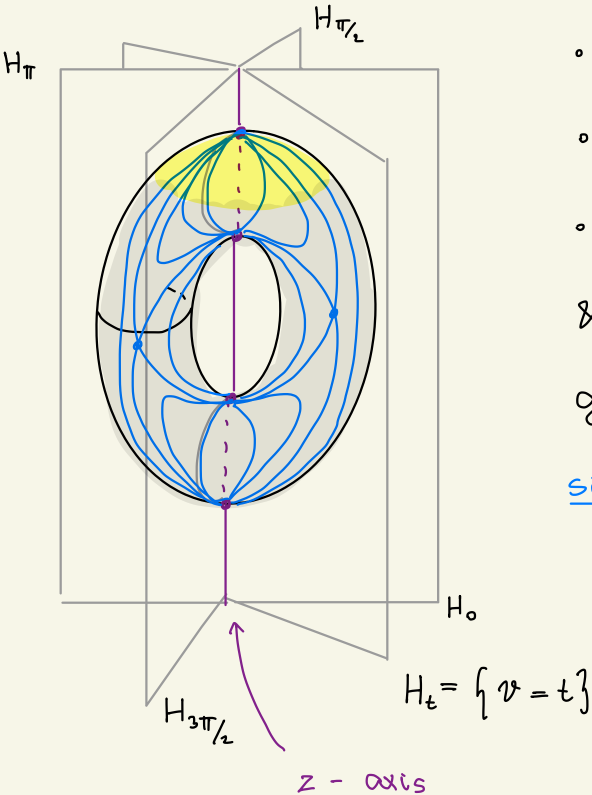

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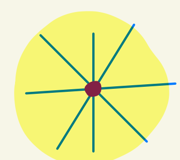
gives a singular foliation on Σ : $\mathcal{F}_{braid}$

singularities:



node

BRAID FOLIATIONS

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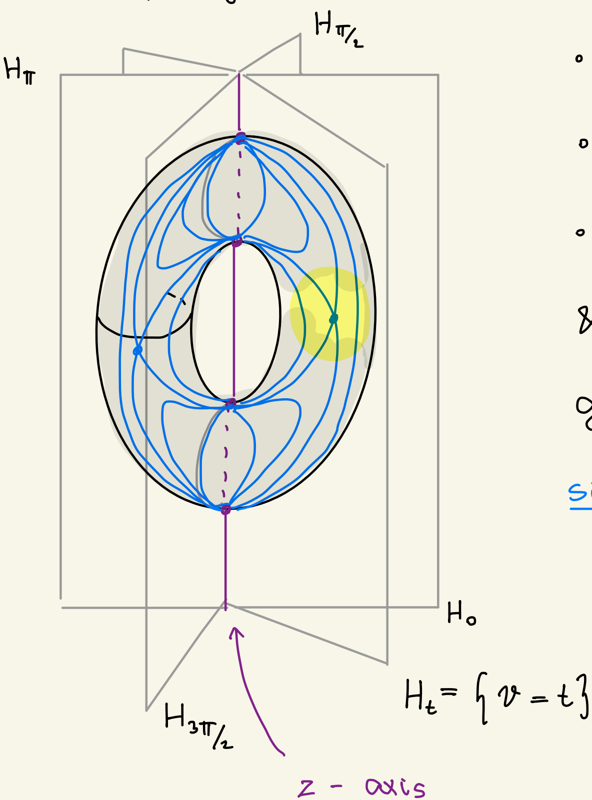

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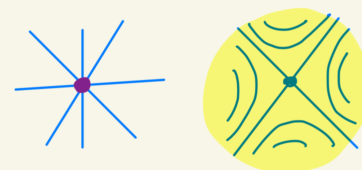

node　　　saddle

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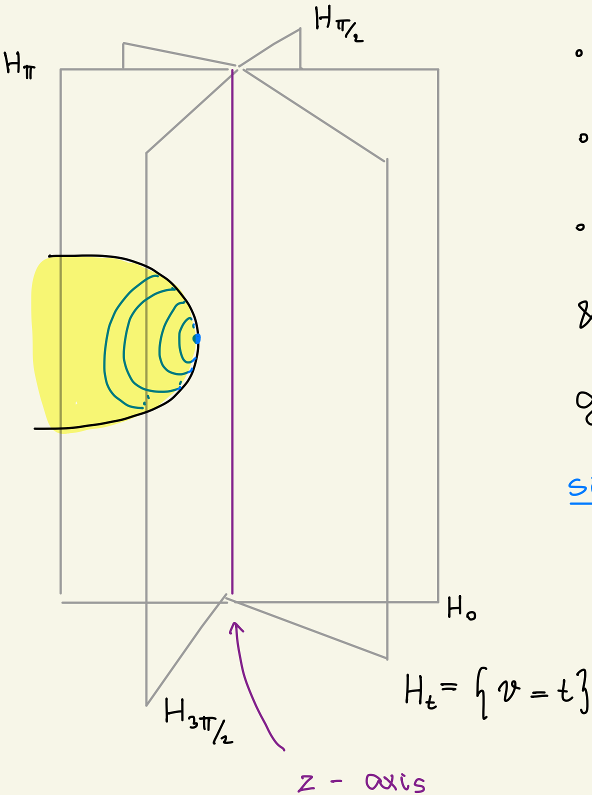

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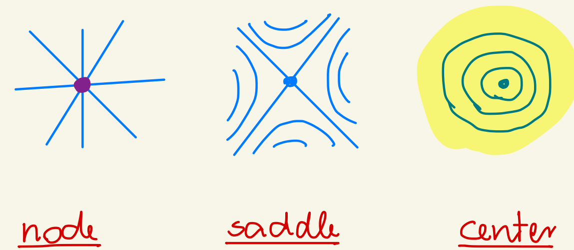
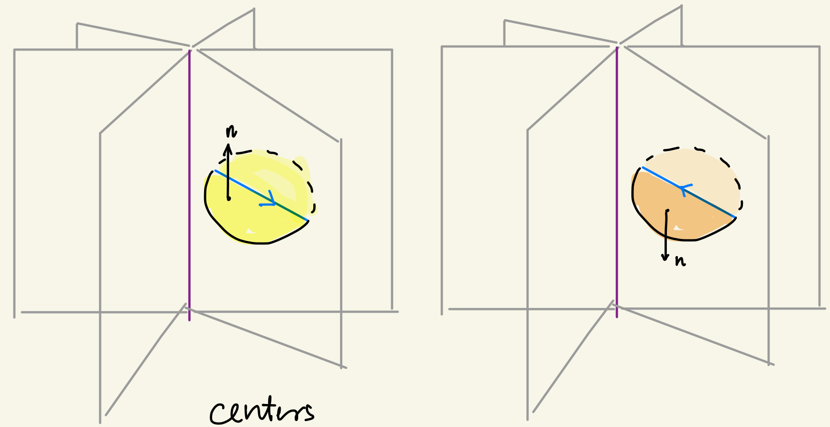


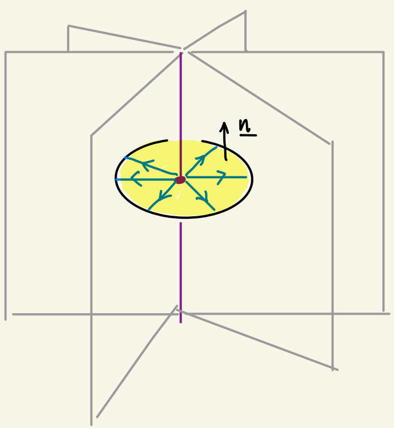
BRAID FOLIATION - SINGULARITIES & ORIENTATION

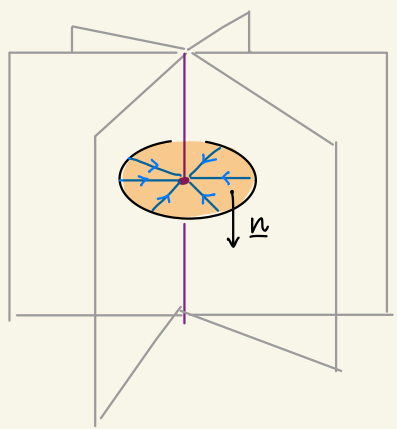
we can orient the leaves:

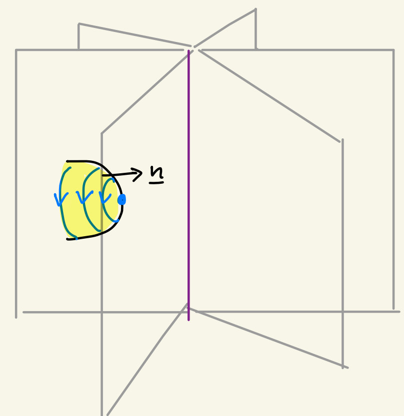


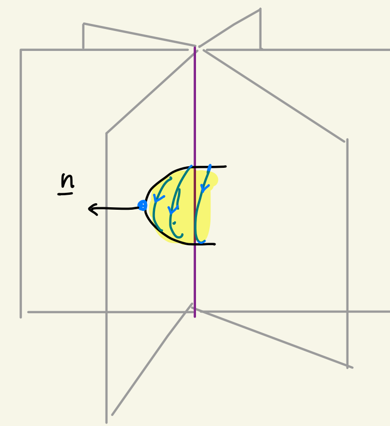
centers

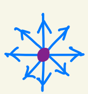
⇒ two types of nodes

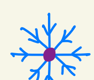




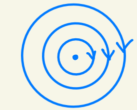




source 

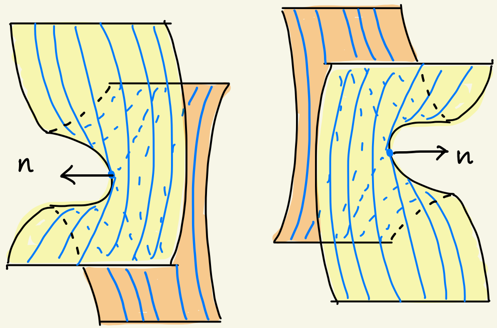
sink 

"max" 

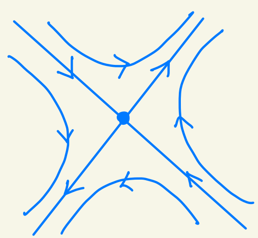
"min" 

BRAID FOLIATION - SINGULARITIES & ORIENTATION

there is also two types of saddles

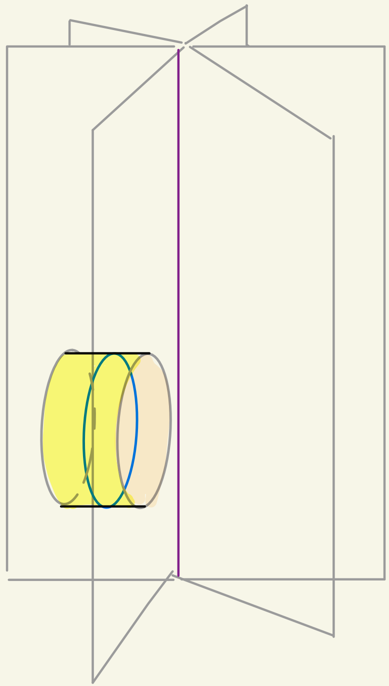


but we cannot distinguish them just from the foliation



BRAID FOLIATION - GETTING RID OF CIRCLES

if we have circles in $\mathcal{F}_{braid}$:



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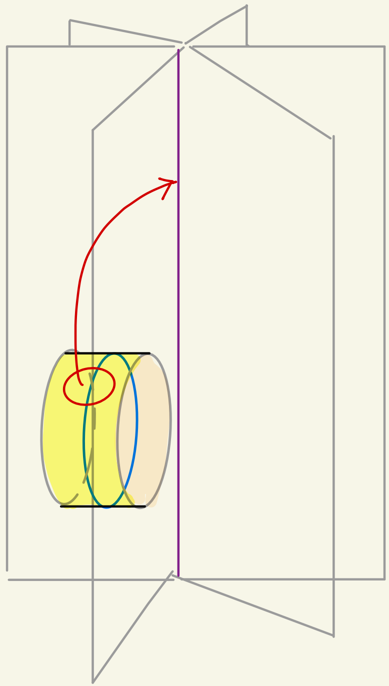

push it through the z-axis

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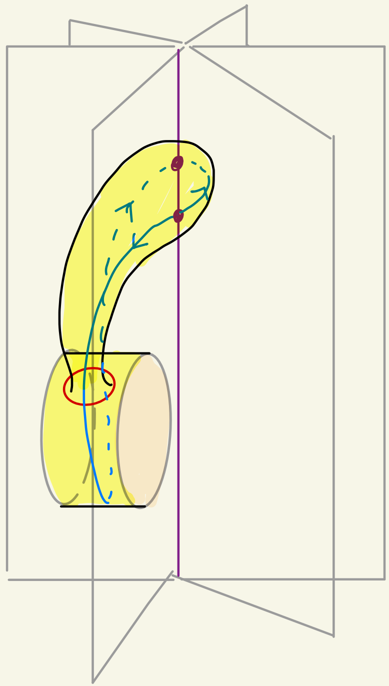
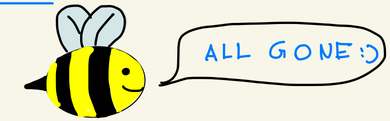


push it through the z-axis

$\Rightarrow$ the circle breaks into pieces

BRAID FOLIATION - GETTING RID OF CIRCLES



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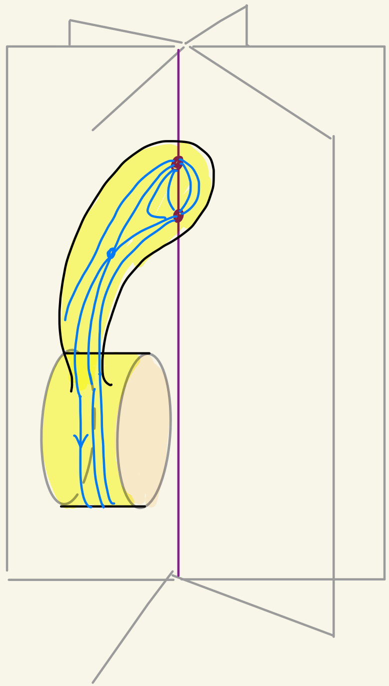
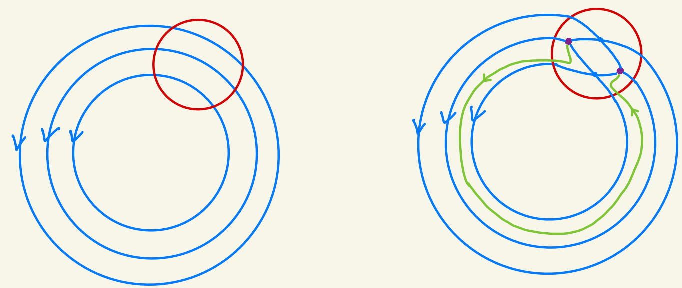


push it through the z-axis

⇒ the circle breaks into pieces

& no more nearby circles are created :



& it kills an interval-worths of circles

⇒ can remove all circles by finitely many finger-moves

WHAT BRAID FOLIATIONS ARE USED FOR ?

- Bennequin 1980 : $(\mathbb{R}^3, \xi = \ker(dz - y\,dx))$ is tight
- Birman - Menasco 1990s : - Markov Theorem w/o stabilisation
  - construction of transversaly nonsimple knots

. . .

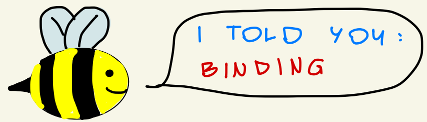
- Dynikov - Prasolov 2018 : computing Legendrian grid number
- LaFountain - Menasco, Dynikov - Prasolov 2013 generalised Jones conjecture

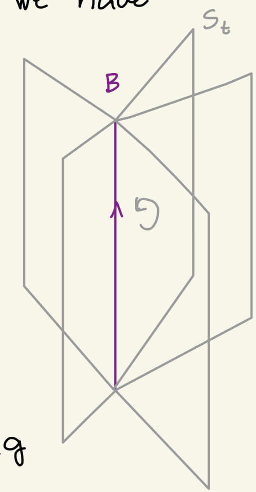
Open Book Decompositions

M^3 $\qquad (B,\pi)$

$B^1 \hookrightarrow M \qquad \pi : M\setminus B \longrightarrow S^1$ fibration , $S_t := \pi^{-1}(t)$ & near B we have

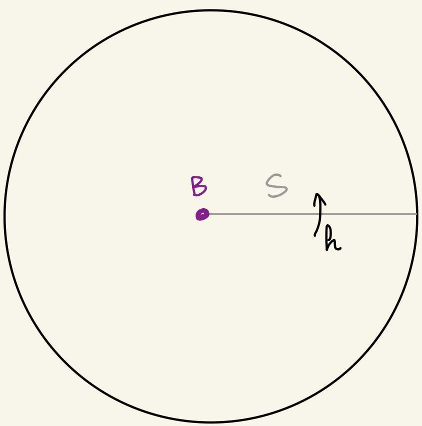
binding $\qquad\qquad$ page





$\Rightarrow$ We can think of $M\setminus B$ as a mapping cylinder of (S,h):

$$S\times I/_{(x,1)\sim(h(x),0)}$$



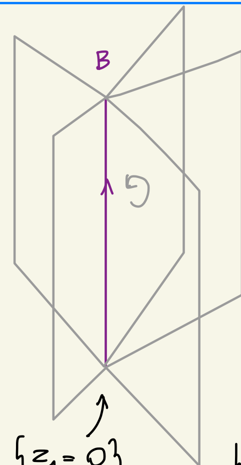
& we obtain M by further identifying

$$(x,t)\sim(x,t') \qquad x\in\partial S,\ t,t'\in I$$

Open Book Decompositions – Examples

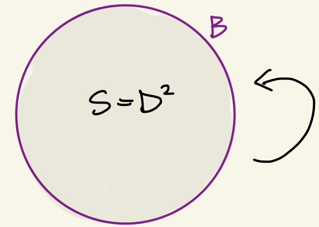
1) $S^3 \subseteq \mathbb{C}^2$
(z_1, z_2)

$$\pi = \frac{z_1}{|z_1|}$$



$\{z_1 = 0\}$

$\rightsquigarrow$

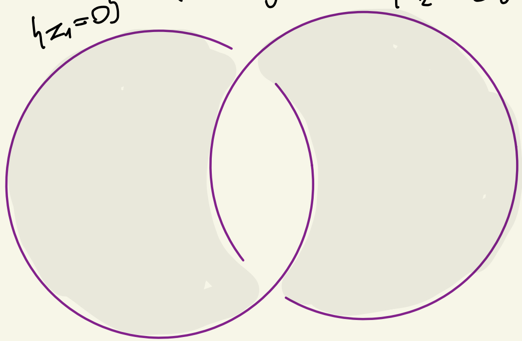


$h = id$

2) $S^3 \subseteq \mathbb{C}^2$

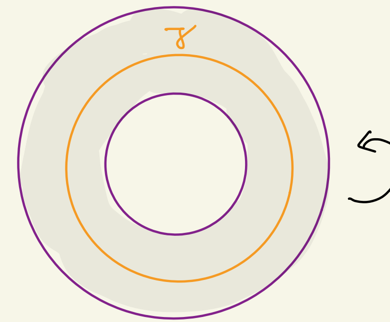
$$\pi = \frac{z_1 z_2}{|z_1 z_2|}$$

$\{z_1 = 0\}$ $\{z_2 = 0\}$



$\rightsquigarrow$

$S =$ annulus



$h = D_\gamma$

Dehn twist along γ

3) $\pi = \dfrac{z_1 \overline{z_2}}{|z_1 \overline{z_2}|}$ (annulus, D_γ^{-1})

Open Book Decompositions – Properties

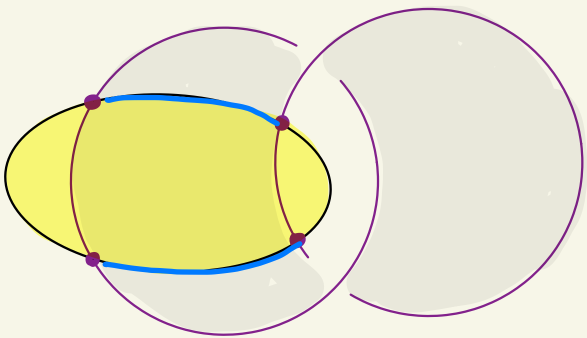
- any 3-manifold admits an open book
  & open books describing the same 3-manifold are related by stabilisation

$$\text{3-manifolds} \longleftrightarrow \text{open book decomposition} \big/ \text{stabilisation}$$

- open books actually describe contact 3-manifolds
  & open books describing the same contact 3-manifold are related by positive stabilisation

$$\text{contact 3-manifolds} \longleftrightarrow \text{open book decomposition} \big/ \text{positive stabilisation}$$

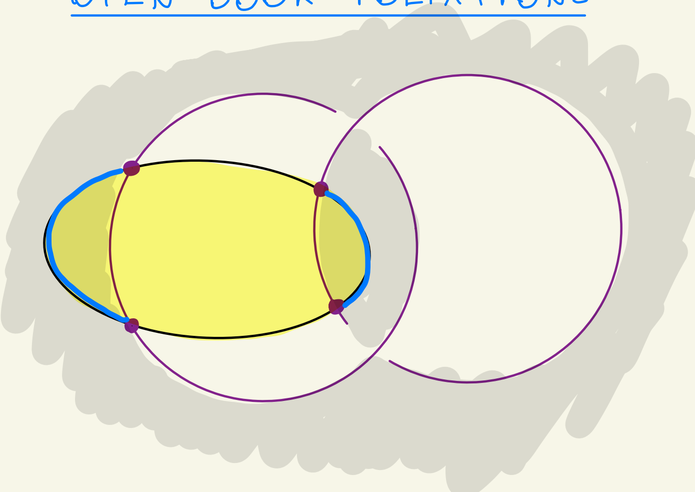
OPEN BOOK FOLIATIONS



- take a surface $\Sigma \hookrightarrow M^3$

& look at its intersection w/ S_t

OPEN BOOK FOLIATIONS



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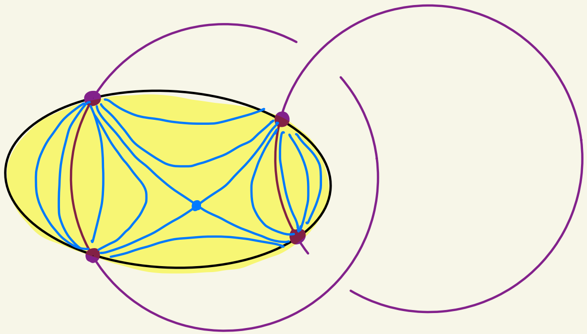

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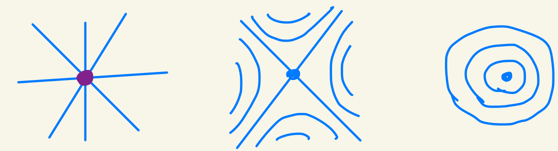
gives a singular foliation on Σ : $\mathcal{F}_{ob}$

singularities:



node saddle center

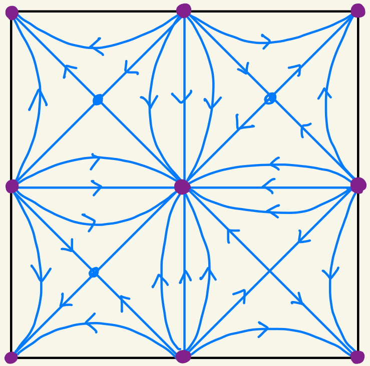
& just as before one can get rid of circle leaves & thus centers

WHAT OPEN BOOK FOLIATIONS ARE USED FOR ?

- Pavelescu: Markov theorem for braids in general open books
- Ito-Kawamuro: - better understanding of open books of overtwisted contact structures
  - computing self-linking number
  - ...
- Licata - V: defined a nicely gluable version of open books for 3-manifolds w/ bdry
- Alishahi - Földvári - Hendricks - Petkova - V: define gluable contact invariant in bordered Floer homology

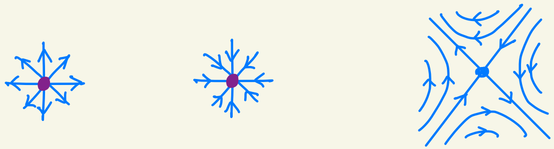
Key: understand open book foliations

OPEN BOOK FOLIATION - PROPERTIES



no circles $\Longrightarrow$ no centers

So: after removing all circles we get
→ oriented singular foliation $\mathcal{F}_{ob}$
on Σ with singularities that
are either nodes or saddles



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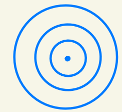
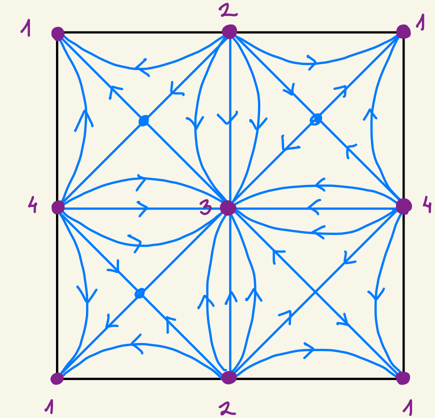
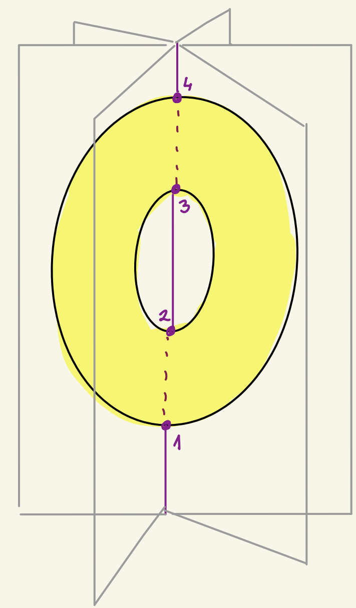
no circles $\Longrightarrow$ no centers 





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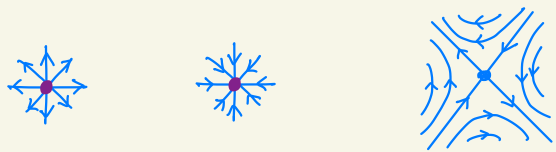

→ the leaves are indexed by a circle

(coming from the S_θ)

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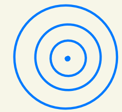
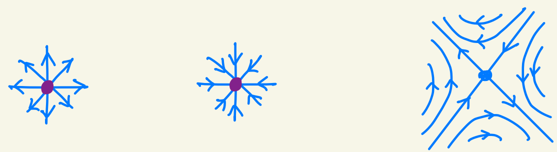
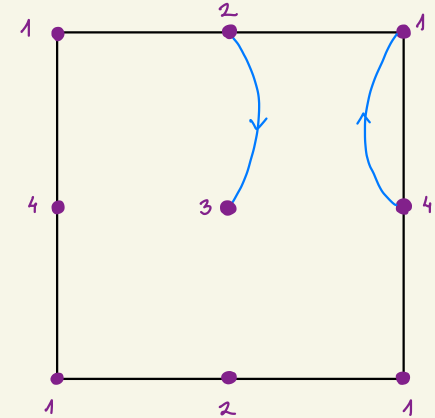
no circles $\Longrightarrow$ no centers 

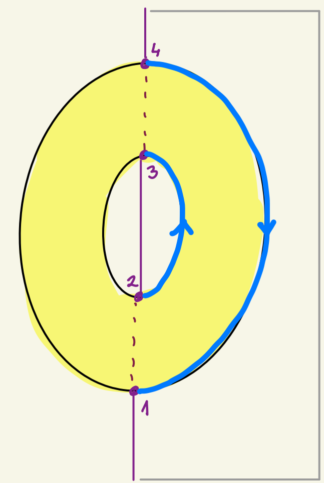
So: after removing all circles we get

→ oriented singular foliation $\mathcal{F}_{ob}$ on Σ with singularities that are either nodes or saddles

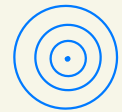


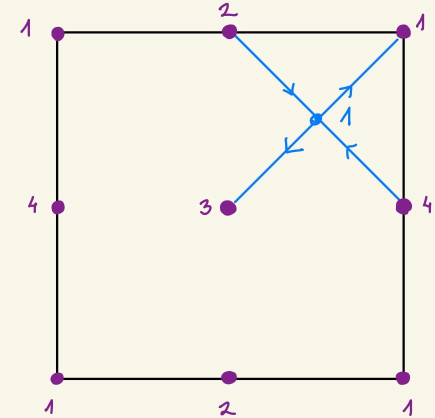
→ the leaves are indexed by a circle (coming from the S_ν)





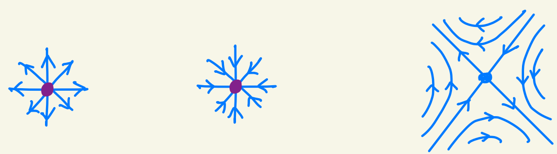
OPEN BOOK FOLIATION – PROPERTIES

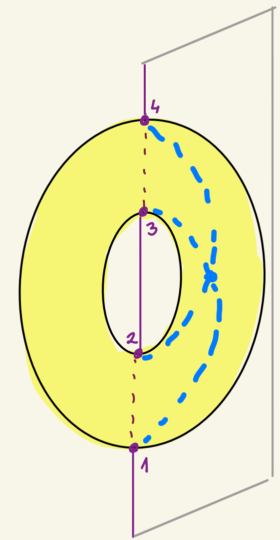
no circles $\Longrightarrow$ no centers 



So: after removing all circles we get

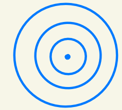
→ oriented singular foliation $\mathcal{F}_{ob}$ on Σ with singularities that are either nodes or saddles

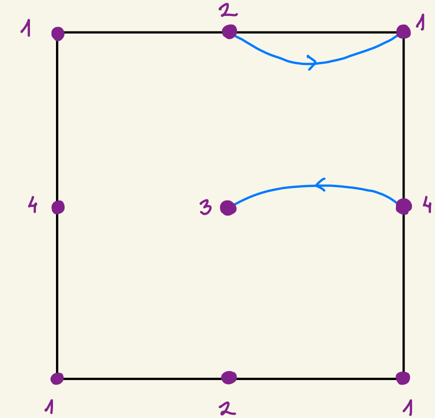




→ the leaves are indexed by a circle (coming from the S_θ)

OPEN BOOK FOLIATION – PROPERTIES

no circles $\Longrightarrow$ no centers 



So: after removing all circles we get

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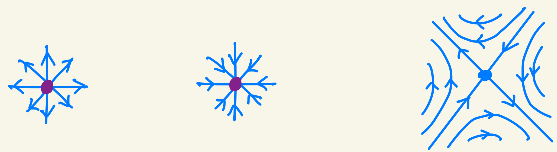
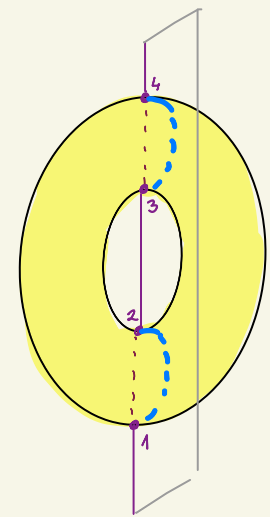
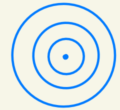




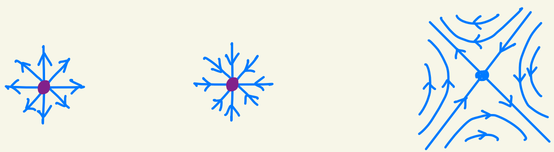
→ the leaves are indexed by a circle (coming from the S_θ)

OPEN BOOK FOLIATION - PROPERTIES

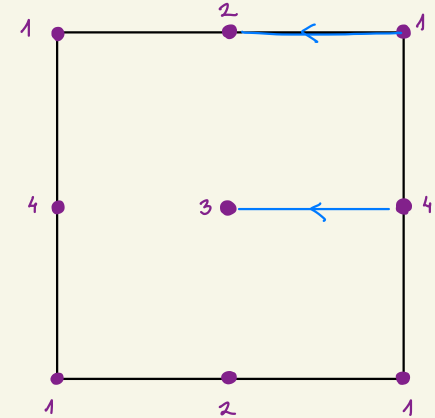
no circles $\Longrightarrow$ no centers 

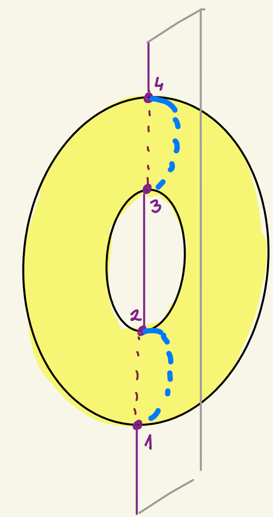
So: after removing all circles we get

→ oriented singular foliation $\mathcal{F}_{ob}$ on Σ with singularities that are either nodes or saddles

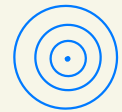


→ the leaves are indexed by a circle (coming from the S_θ)



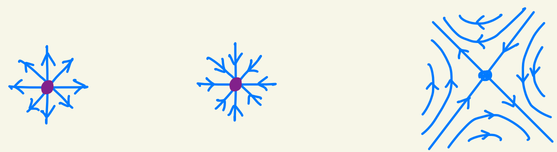


OPEN BOOK FOLIATION - PROPERTIES

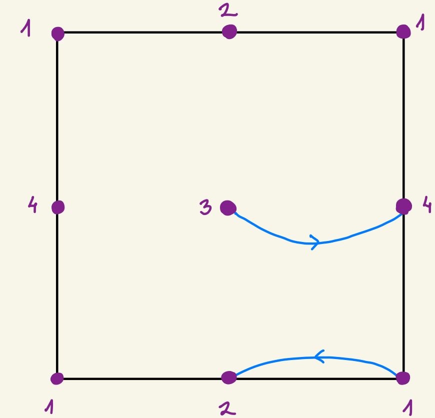
no circles $\Longrightarrow$ no centers 

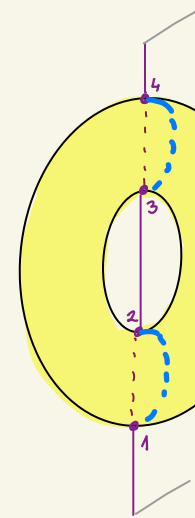
So: after removing all circles we get

→ oriented singular foliation $\mathcal{F}_{ob}$ on Σ with singularities that are either nodes or saddles

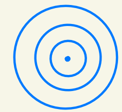


→ the leaves are indexed by a circle (coming from the S_ν)



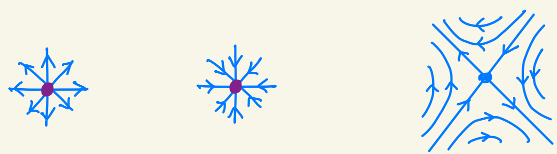


OPEN BOOK FOLIATION - PROPERTIES

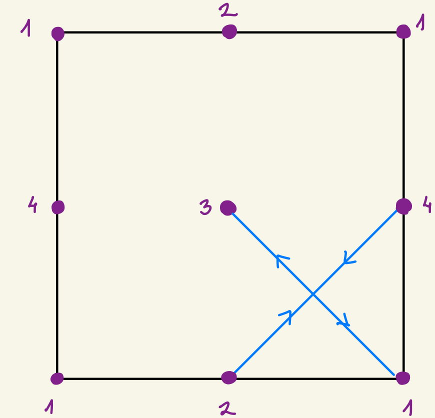
no circles $\Longrightarrow$ no centers 

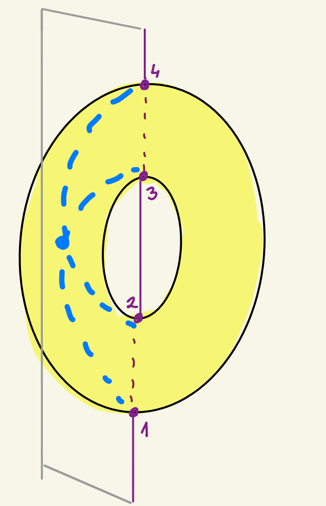
So: after removing all circles we get

→ oriented singular foliation $\mathcal{F}_{ob}$ on Σ with singularities that are either nodes or saddles

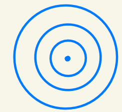


→ the leaves are indexed by a circle (coming from the S_ϑ)



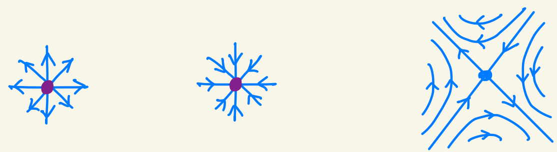


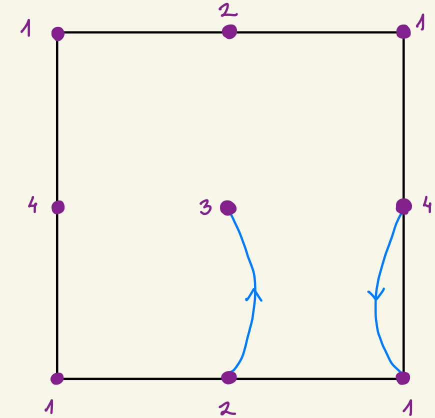
OPEN BOOK FOLIATION - PROPERTIES

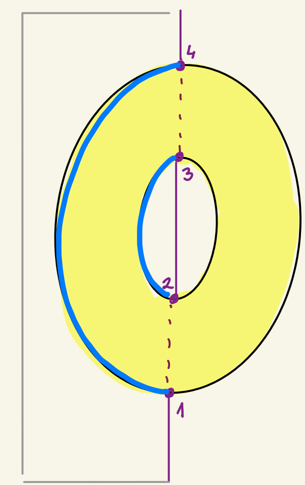
no circles $\Longrightarrow$ no centers 

So: after removing all circles we get

→ oriented singular foliation $\mathcal{F}_{ob}$ on Σ with singularities that are either nodes or saddles

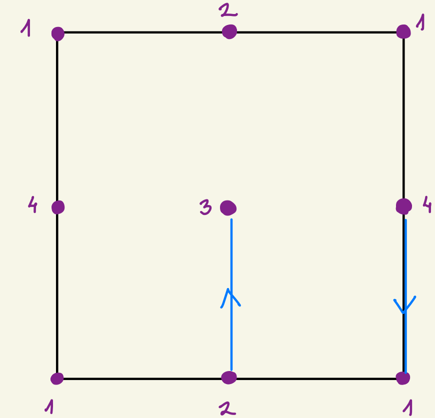


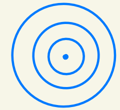




→ the leaves are indexed by a circle (coming from the S_v)

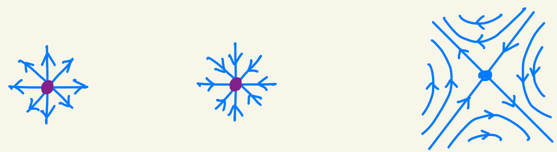
OPEN BOOK FOLIATION - PROPERTIES

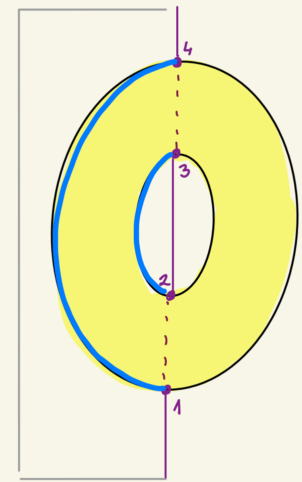


no circles $\Longrightarrow$ no centers 

So: after removing all circles we get

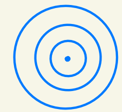
→ oriented singular foliation $\mathcal{F}_{ob}$ on Σ with singularities that are either nodes or saddles





→ the leaves are indexed by a circle (coming from the S_θ)

OPEN BOOK FOLIATION - PROPERTIES

no circles $\Longrightarrow$ no centers 

So: after removing all circles we get

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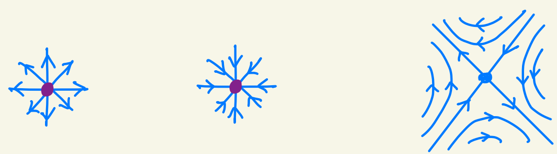
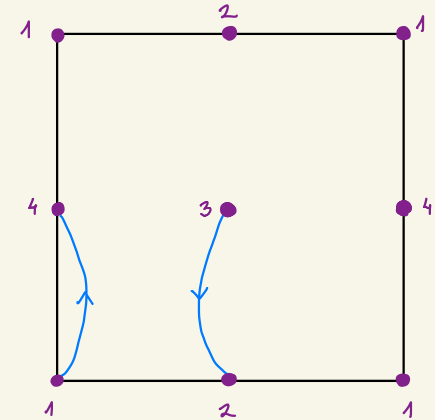
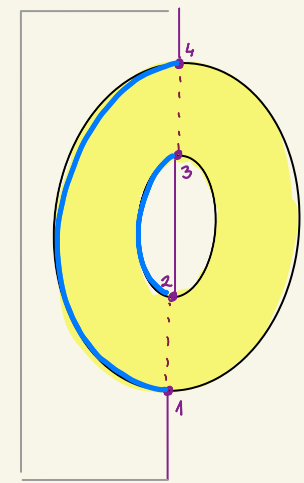
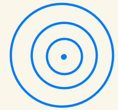


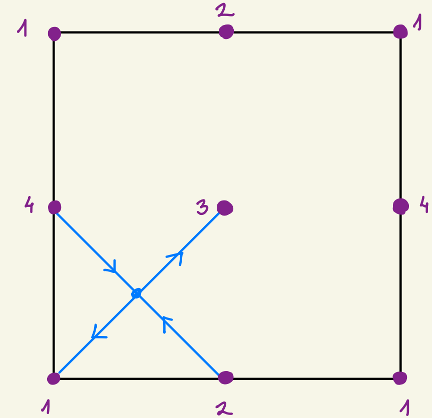
→ the leaves are indexed by a circle (coming from the S_θ)





OPEN BOOK FOLIATION - PROPERTIES

no circles $\Longrightarrow$ no centers 



So: after removing all circles we get

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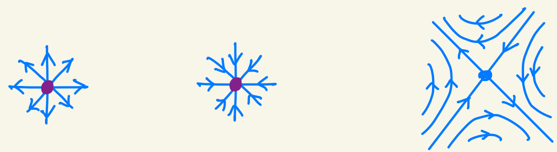
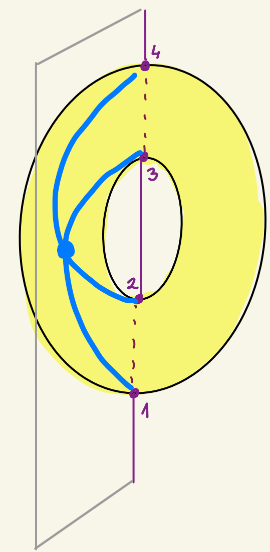
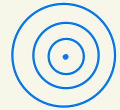




→ the leaves are indexed by a circle (coming from the S_θ)

OPEN BOOK FOLIATION - PROPERTIES

no circles $\Longrightarrow$ no centers 

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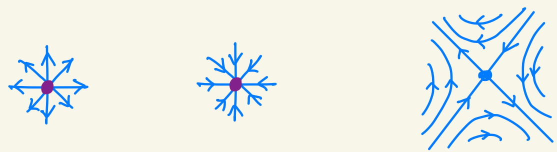
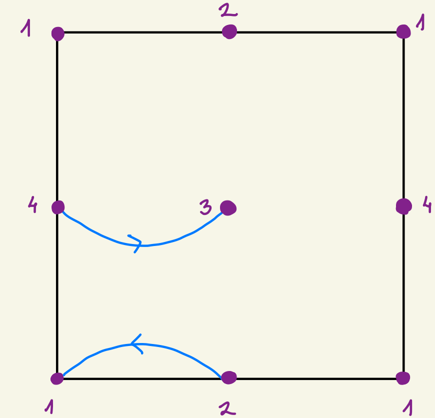
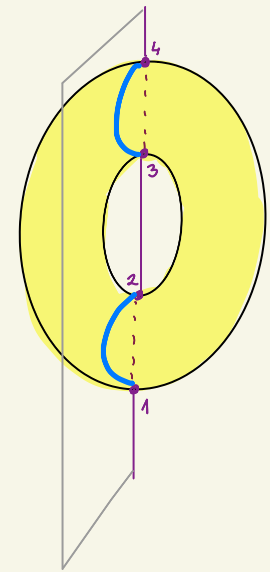


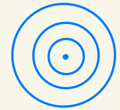
→ the leaves are indexed by a circle (coming from the S_φ)

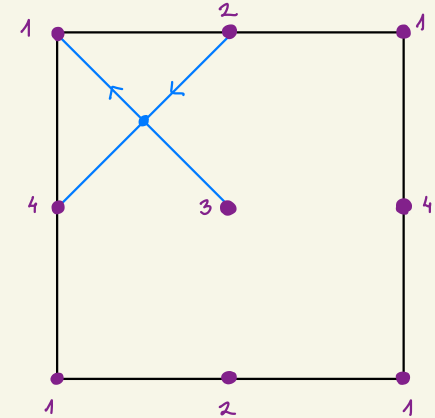




OPEN BOOK FOLIATION - PROPERTIES

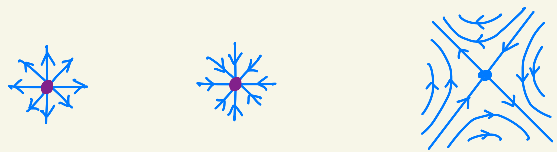
no circles $\Longrightarrow$ no centers

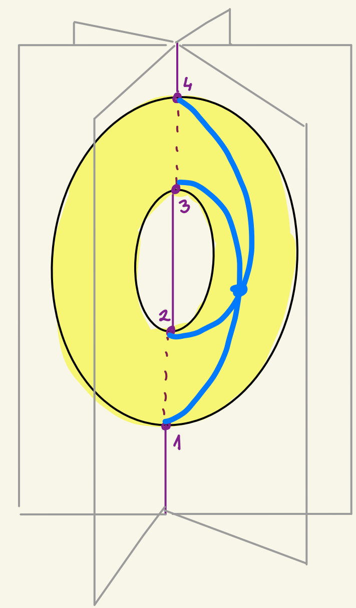




<u>So</u>: after removing all circles we get

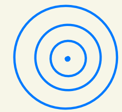
→ oriented singular foliation $\mathcal{F}_{braid}$ on Σ with singularities that are either nodes or saddles

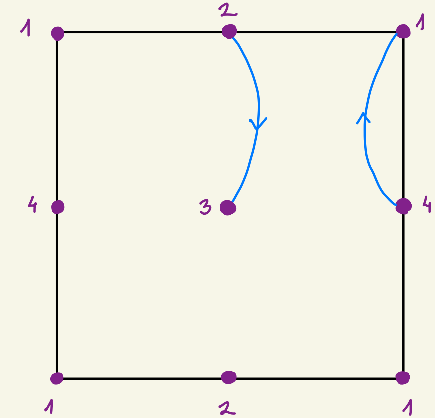




→ the leaves are indexed by a circle (coming from the S_θ)

OPEN BOOK FOLIATION - PROPERTIES

no circles $\Longrightarrow$ no centers 



So: after removing all circles we get

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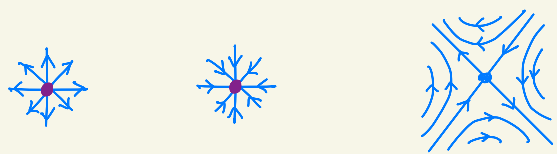
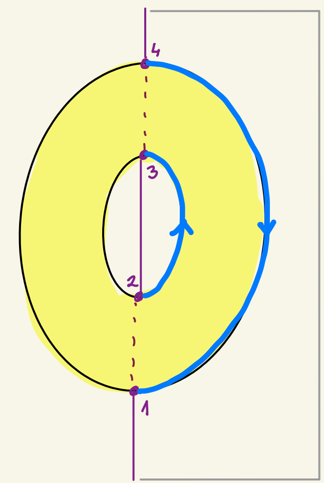
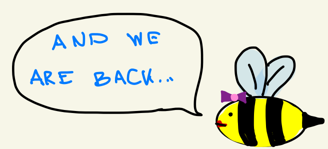
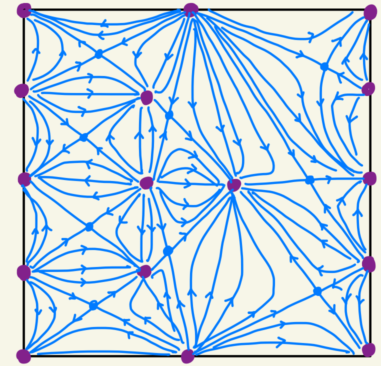




→ the leaves are indexed by a circle (coming from the S_ν) &



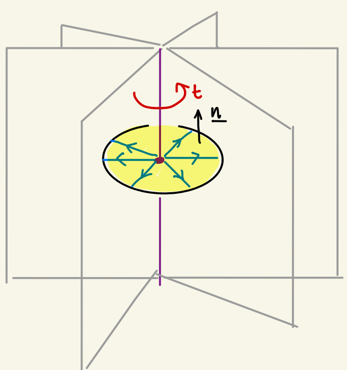
RECOGNISING OPEN BOOK FOLIATIONS

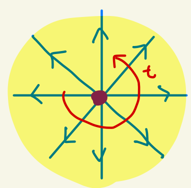


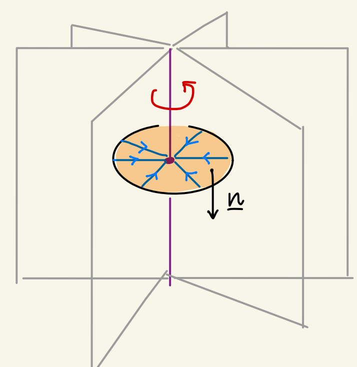
Given: $\mathcal{F}$ oriented singular foliation on Σ with singularities that are either nodes or saddles

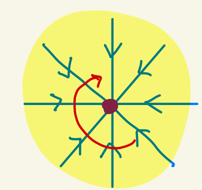
Question: Is $\mathcal{F}$ an open book foliation?

Necessary: leaves are indexed by the circle, so that near each node the indices go around once:



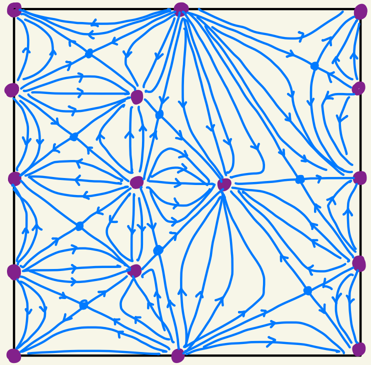






from the other side

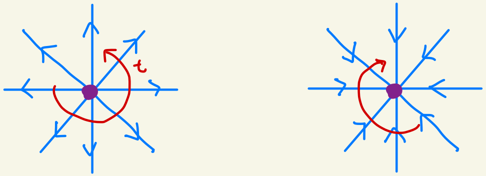
RECOGNISING OPEN BOOK FOLIATIONS



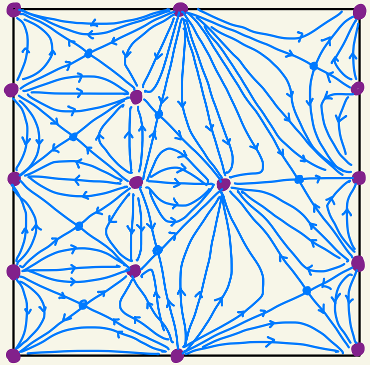
Question: Is $\mathcal{F}$ an open book foliation?

(seemingly)

Simpler Question Can the leaves be indexed by the circle, so that near each node the indices go around once:



RECOGNISING OPEN BOOK FOLIATIONS

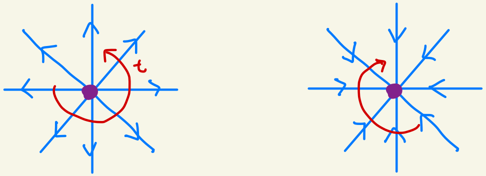


Question: Is $\mathcal{F}$ an open book foliation?

⇕ Construction (Licata – V)

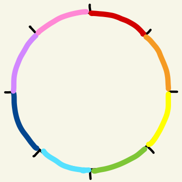
(seemingly)

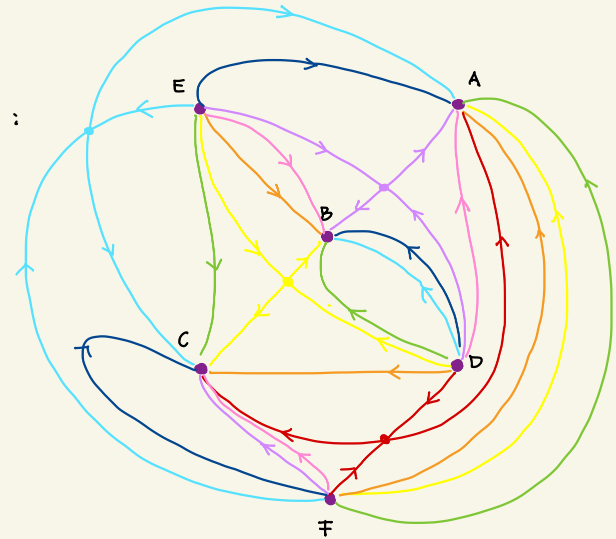
Simpler Question Can the leaves be indexed by the circle, so that near each node the indices go around once:



RECOGNISING OPEN BOOK FOLIATIONS

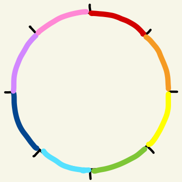
So this must be an open book foliation:

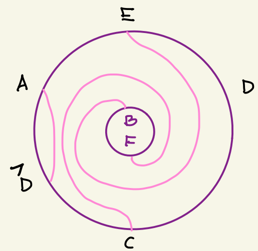


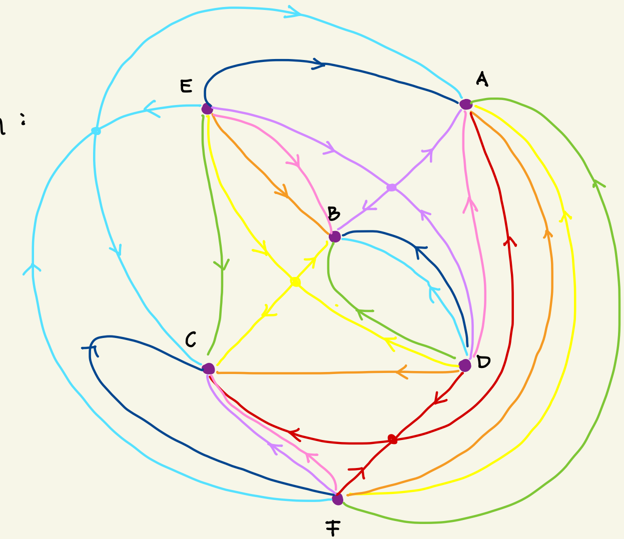


RECOGNISING OPEN BOOK FOLIATIONS

So this must be an open book foliation:

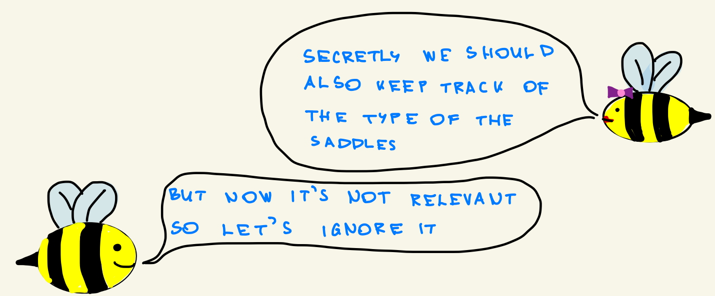






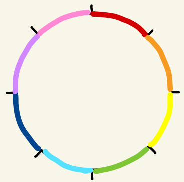
indeed:

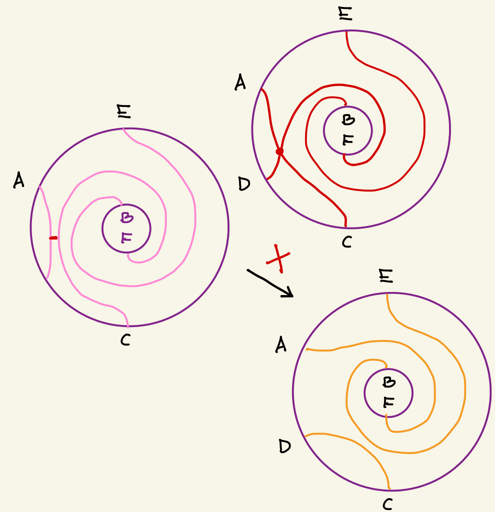
$\rightsquigarrow \; S^2 \hookrightarrow S^3$

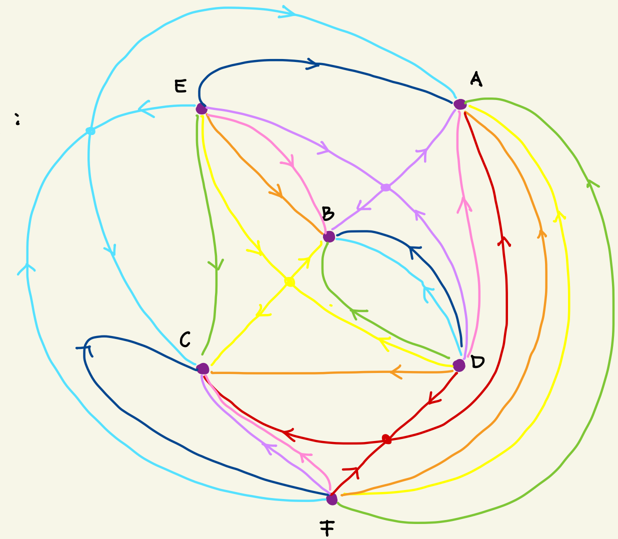


RECOGNISING OPEN BOOK FOLIATIONS

So this must be an open book foliation:







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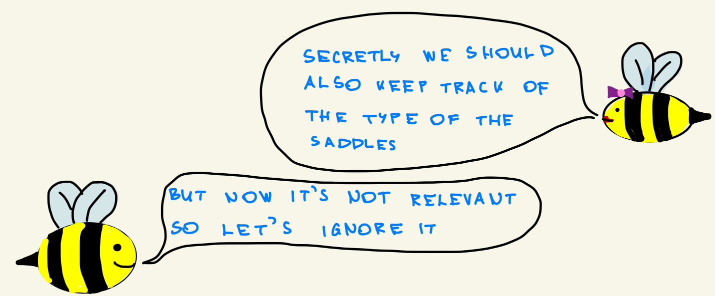

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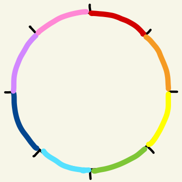

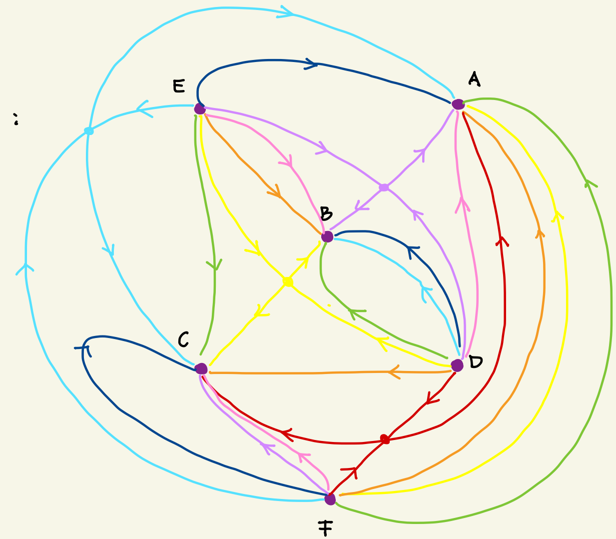

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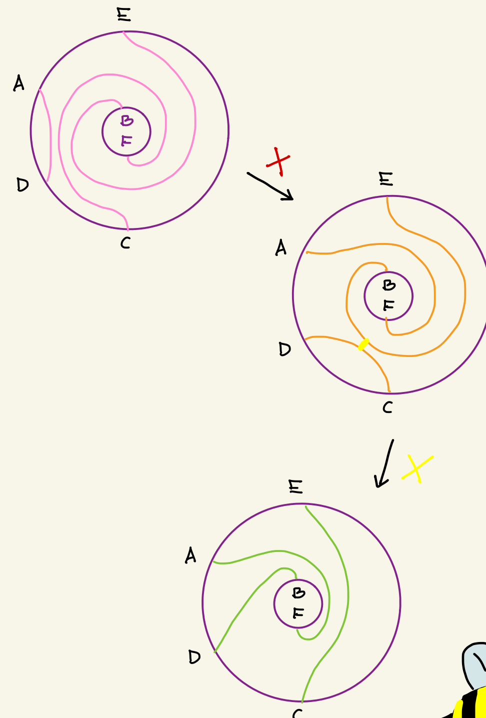

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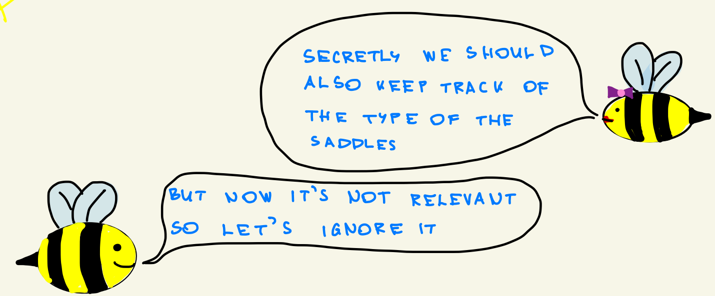

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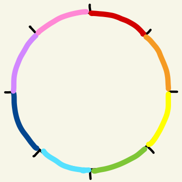

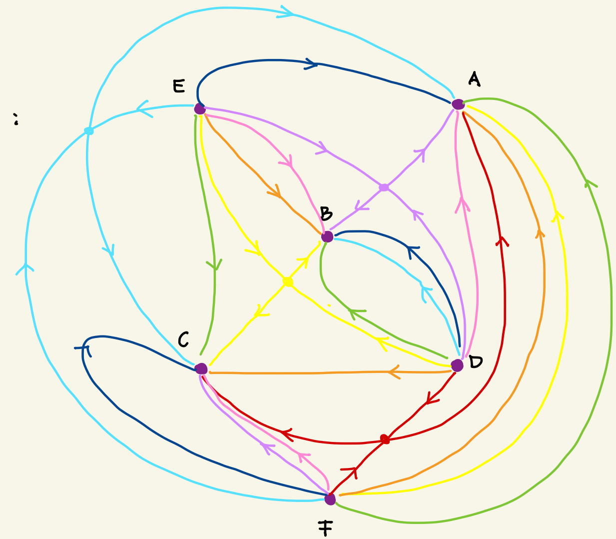

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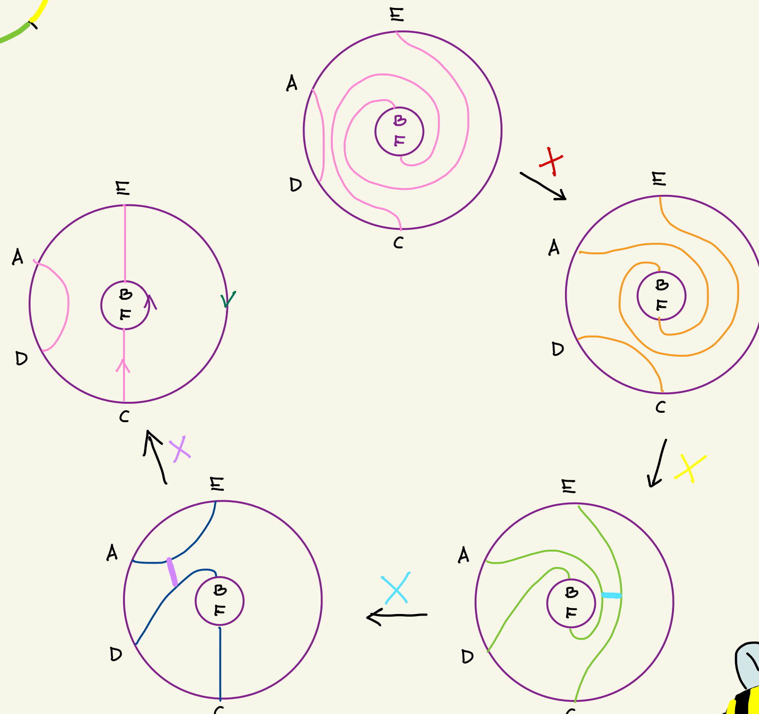

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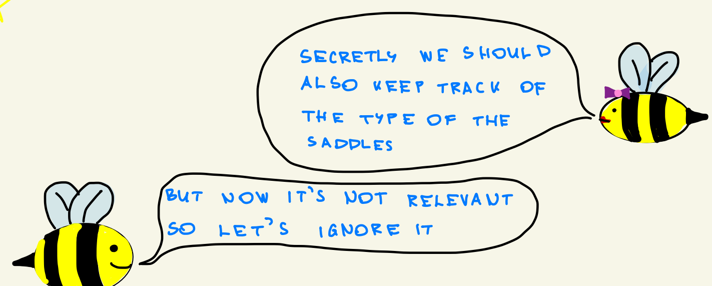

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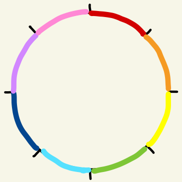

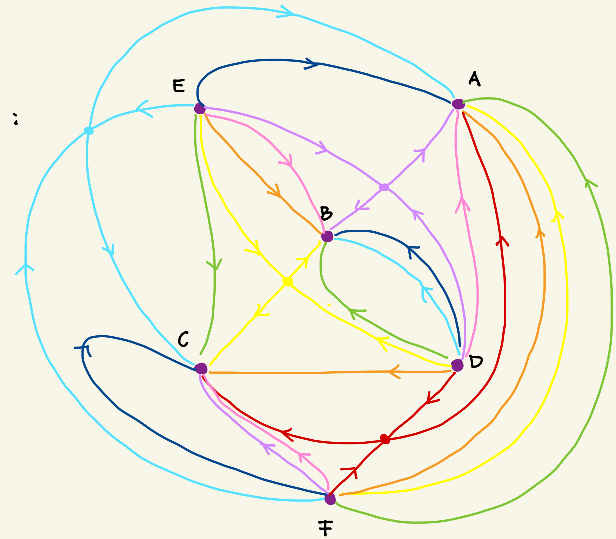

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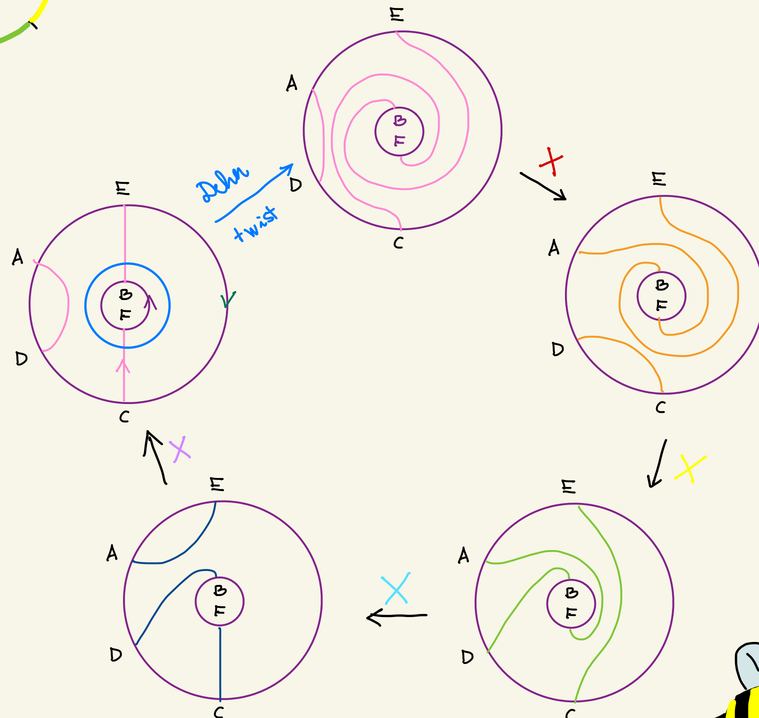

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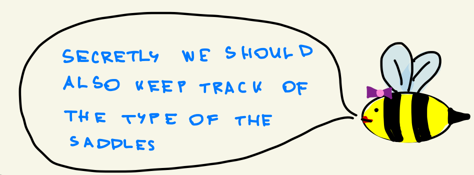
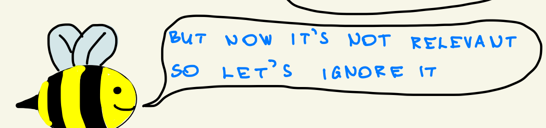




RECOGNISING OPEN BOOK FOLIATIONS - COMPLEXITY

- it is enough to find a cyclic order of the saddles.

  (the order can then be extended to all the leaves)

- each node gives a partial cyclic order of the saddles

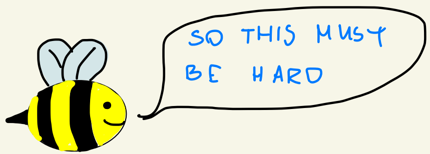
  & extending partial cyclic orders to a (total) cyclic order

  is NP-complete in general (Galil-Megdido 1977)

- the problem can be rephrased to finding maximal acyclic subgraph

  of an oriented graph D associated to $\mathcal{F}$

  & these are NP-complete in general (Karp 1972)



- the problem can be rephrased to finding the minimal

  genus surface D can be embedded into

  & these are NP-complete in general (Thomasse 1988)

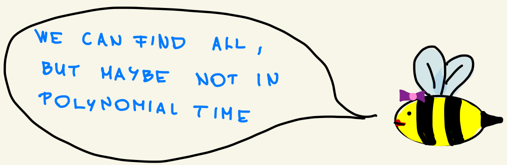
RECOGNISING OPEN BOOK FOLIATIONS - ALGORITHM

Thm (Kiss - V): There is a polynomial algorithm that recognises whether a given oriented singular foliation $\mathcal{F}$ is an open book foliation.

(= there is a "good" cyclic order for the saddles)

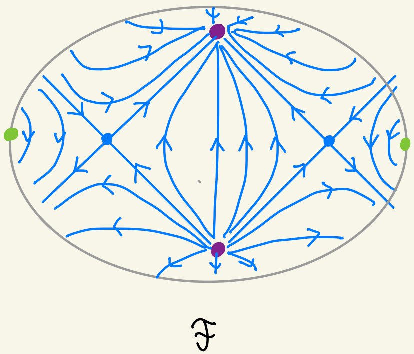
Morover if there is such a cyclic order, then it can be found in polynomial time.

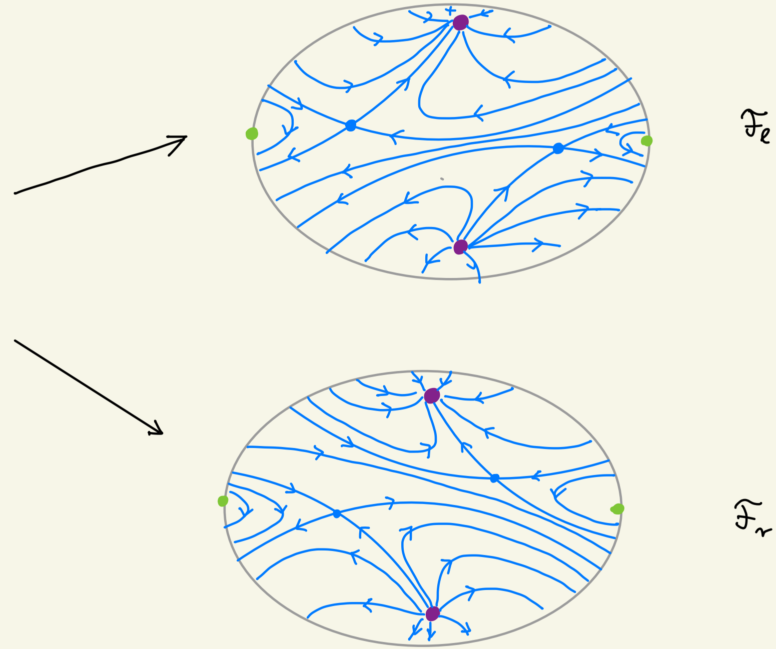
Rmk: usually there is more than one cyclic order, & the algorithm only finds one of them.

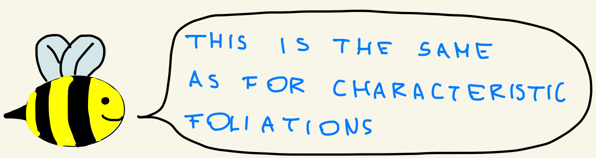


RECOGNISING OPEN BOOK FOLIATIONS – IDEA

Bypass - move :







Lemma 1 Unless $\mathcal{F}_\ell$ or $\mathcal{F}_r$ has circle leaves we have

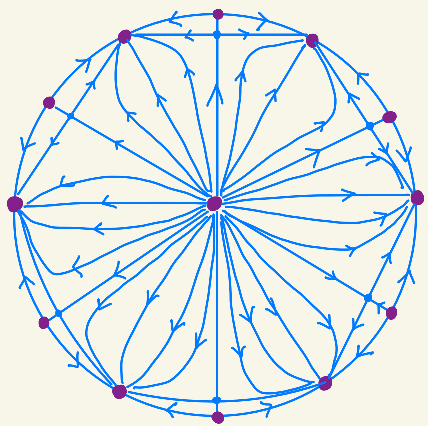
$\mathcal{F}$ is an OBF $\Leftrightarrow$ $\mathcal{F}_r$ is an OBF $\Leftrightarrow$ $\mathcal{F}_\ell$ is an OBF

RECOGNISING OPEN BOOK FOLIATIONS – IDEA

Lemma 1 Unless $\mathcal{F}_\ell$ or $\mathcal{F}_r$ has circle leaves we have

$$\mathcal{F} \text{ is an OBF} \Leftrightarrow \mathcal{F}_r \text{ is an OBF} \Leftrightarrow \mathcal{F}_\ell \text{ is an OBF}$$

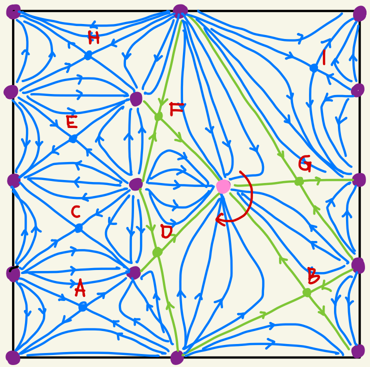
Lemma 2: By a finite sequence of bypass moves we can turn any foliation into one where $\exists$ node w/ a leaf to EVERY saddle:



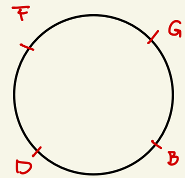
(and the edges are idenified in pairs)

& here it's easy to tell if its an OBF :)

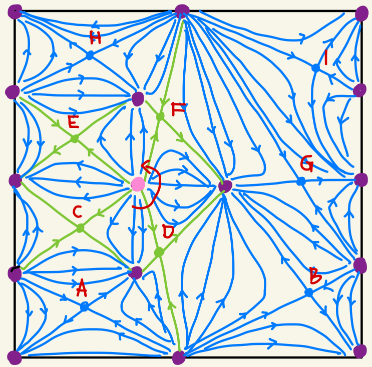
RECOGNISING OPEN BOOK FOLIATIONS – EXAMPLE



We need a cyclic order that extends:



RECOGNISING OPEN BOOK FOLIATIONS – EXAMPLE



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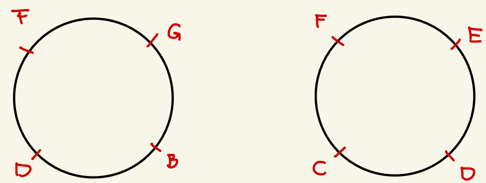
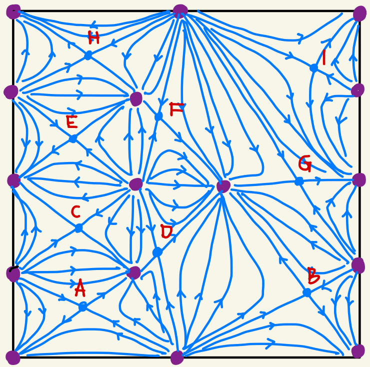


RECOGNISING OPEN BOOK FOLIATIONS – EXAMPLE



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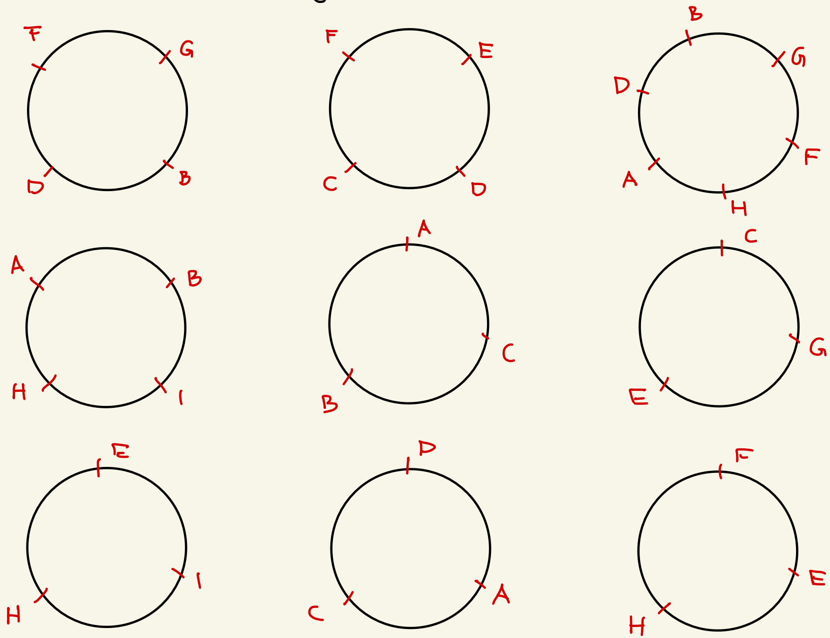
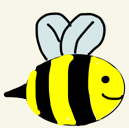
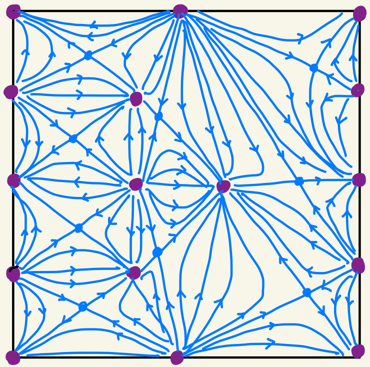




CAN YOU FIND ONE?

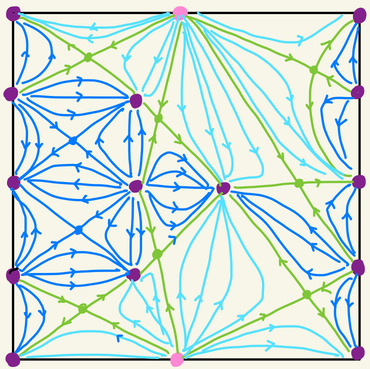
LET'S RUN THE ALGORITM INSTEAD

RECOGNISING OPEN BOOK FOLIATIONS - EXAMPLE



9 nodes , 9 saddles

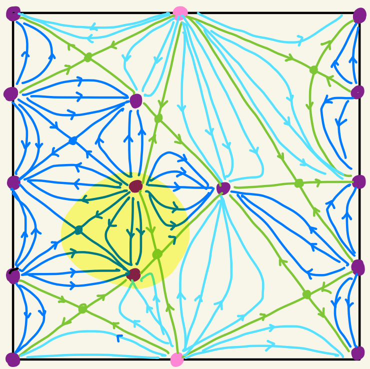
RECOGNISING OPEN BOOK FOLIATIONS - EXAMPLE



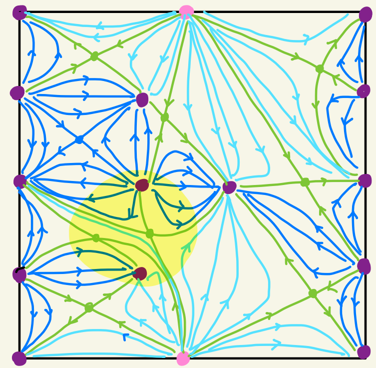
9 nodes, 9 saddles

connected to 7 saddles, let's increase !

RECOGNISING OPEN BOOK FOLIATIONS – EXAMPLE



$\rightarrow$



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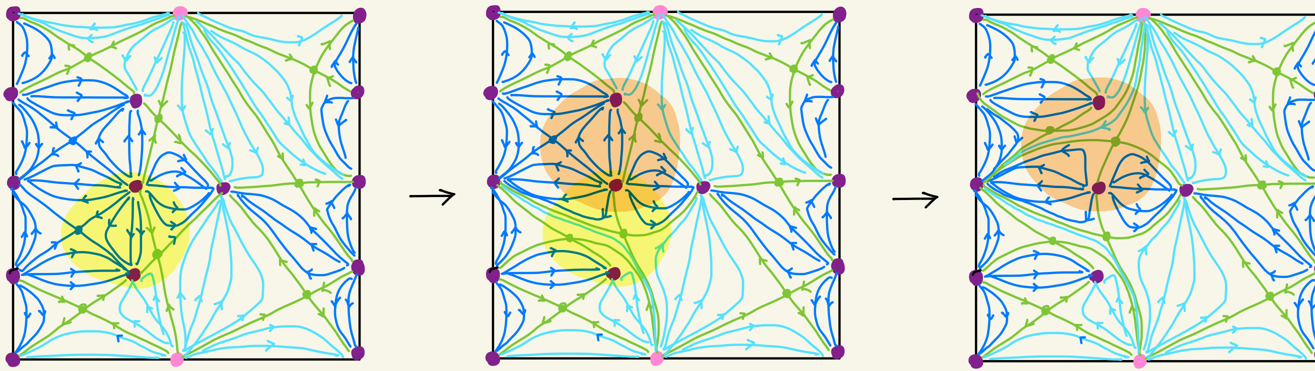

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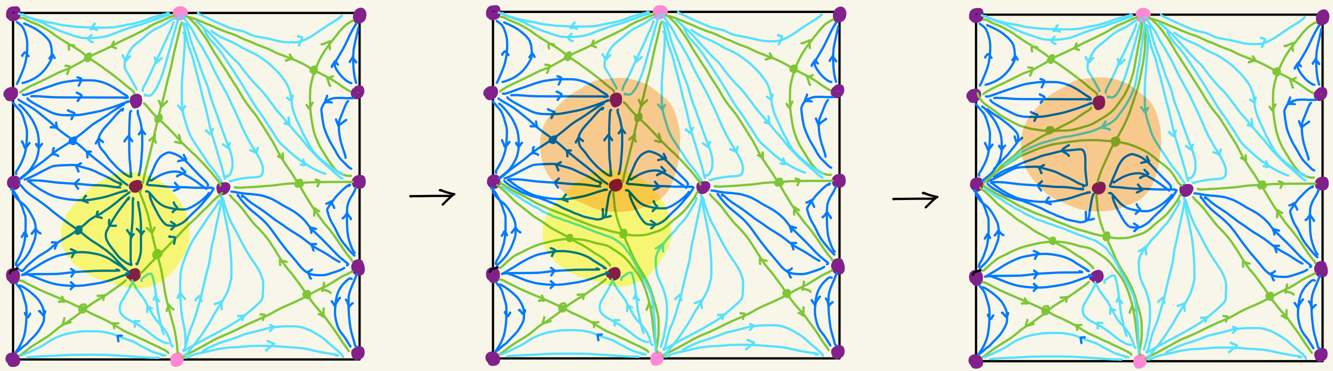
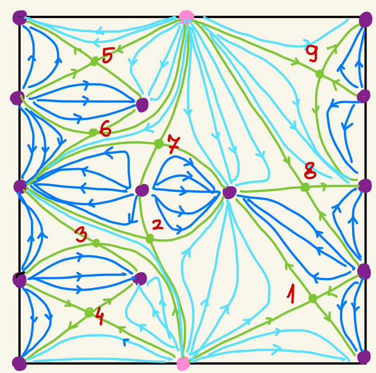


9 nodes, 9 saddles

$\Rightarrow$ only 1 possible cyclic order



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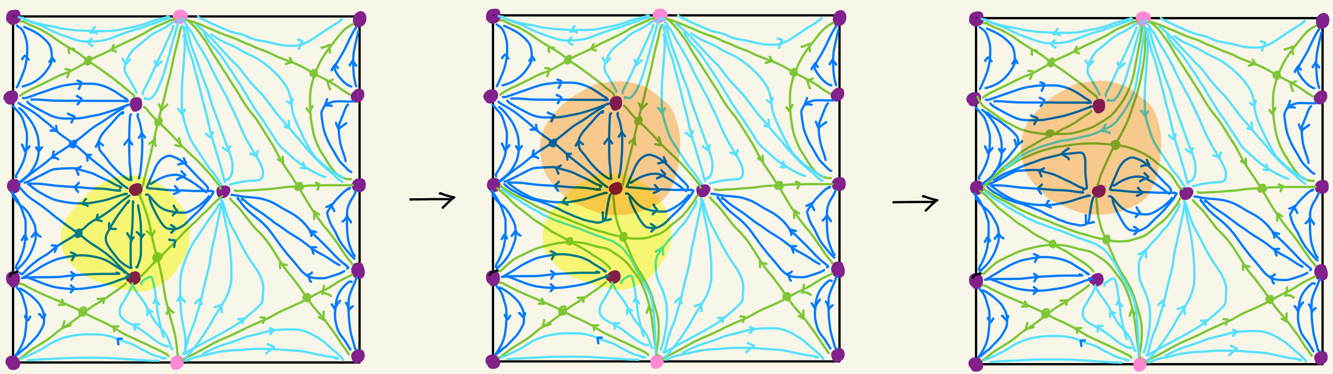
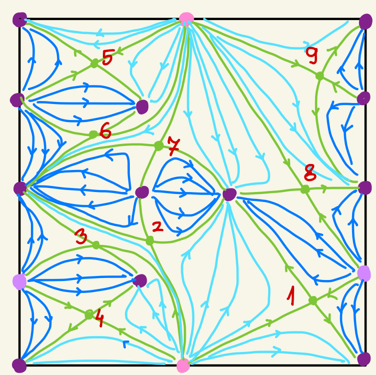
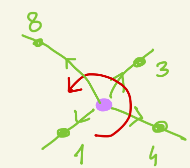


9 nodes, 9 saddles

⇒ only 1 possible cyclic order:

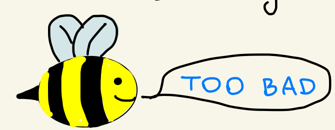


but: the order around this node is



doesn't go around once

So $\mathcal{F}$ was NOT an open book foliation!



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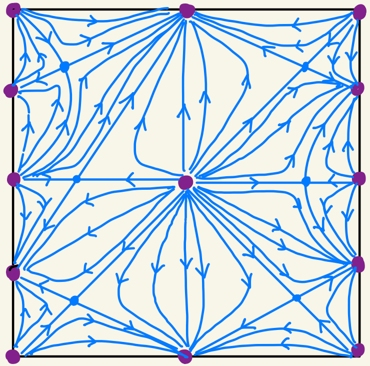

6 nodes , 6 saddles

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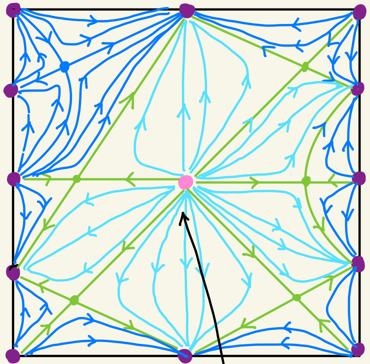
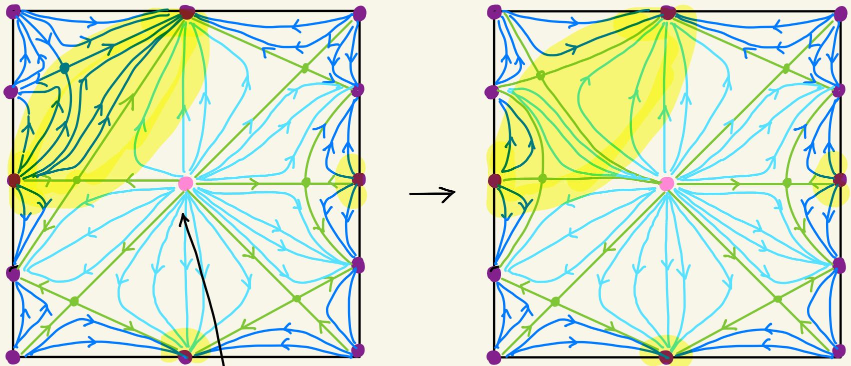


6 nodes , 6 saddles

connected to 5 saddles

RECOGNISING OPEN BOOK FOLIATIONS – EXAMPLE



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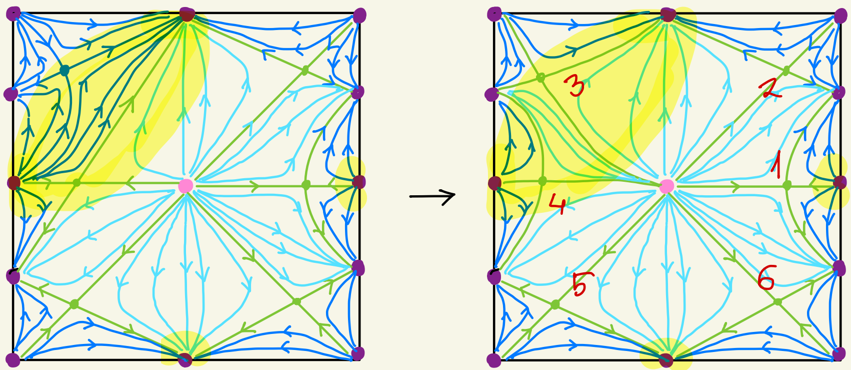

unique cyclic order

& it is "good"

6 nodes, 6 saddles

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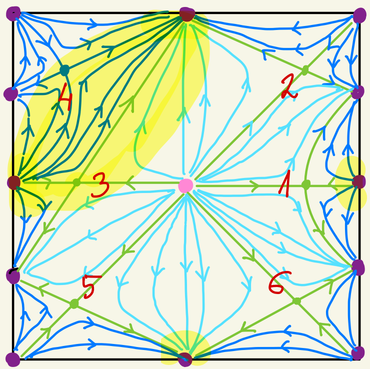

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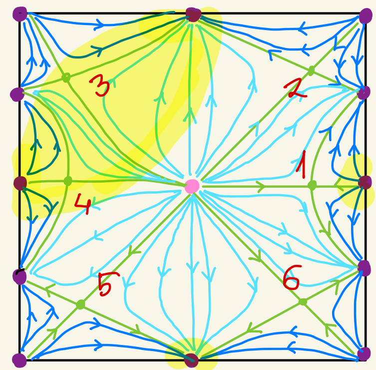


unique cyclic order

& it is "good"

6 nodes, 6 saddles

getting back the "good" order

Rmk: the change in the cyclic order does not have to be "local", it can change even between saddles that do not move.

OPEN QUESTIONS

- Can one recognise braid foliations?

  (done for: $\Sigma = S^2$ or T^2)

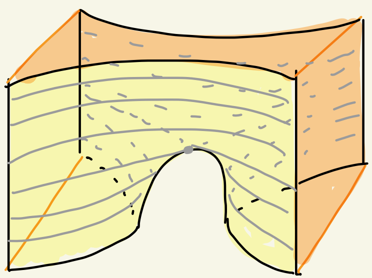
- Can one recognise open book foliations coming from embeddings into a fixed open book?

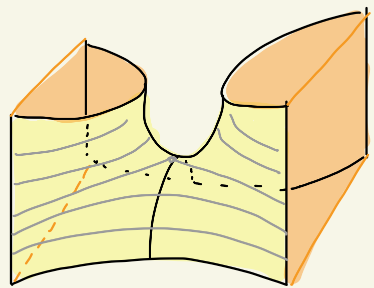
- Can one enumerate all possible embeddings giving the same open book foliation?

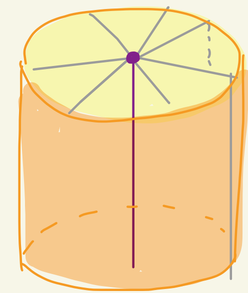
  (for braid foliations, once the order of saddles is fixed there is a unique embedding (Birman-Finkelstein 1998))

THANKS FOR 

YOUR ATTENTION!







QUESTIONS ?