Math 4432 - Spring 2019
Homework 1

Work all these problems and talk to me if you have any questions on them, but carefully write up and turn in **only** *problems 1, 2, 4, 5, 6, 8, 9, and 10.* **Due: January 23**

1. Show the figure eight knot is not 3 colorable.

2. Compute the Alexander polynomial of the figure eight knot.

3. Implicit in much of our discussion of computing the Alexander and Jones polynomials of a knot is that starting with any diagram you can change some of the crossings to get a diagram of the unlink. Another way to think of this is that if you have a diagram for a knot (an analogous result is true for links too) such that as you travel along the knot the first time you come to a crossing it is always over the other strand, then the knot is the unknot. Prove this.

4. Compute $\Delta_K(1)$. Warning, the answer depends on the number of components K has. Hint: think about the skein relation.

5. Compute the Jones polynomial of the mirror of the trefoil (that is the left handed trefoil) using the skein relation.

6. Compute the Jones polynomial of the trefoil (the right handed trefoil) using the Kauffman bracket definition.

7. Compute the Jones polynomial of the figure eight knot using the Kauffman bracket definition.

8. Show that the bracket polynomial (in the variable A, B, and d) satisfies

$$\langle D \amalg U \rangle = d\langle D \rangle,$$

   where $D \amalg U$ is the link diagram consisting of D together with a disjoint unknot.

9. Show that $F_{m(K)}(A) = F_K(A^{-1})$, recall $m(K)$ is the mirror image of K.

10. Show that $F_{\overline{K}}(A) = F_K(A)$, recall $\overline{K}$ means K with the opposite orientation.

11. Show that for any knot K, $V_K(1) = (-2)^{|K|}$, recall that $|K|$ denotes the number of components of K.