Math 4432 - Spring 2019
Homework 4

Work all these problems and talk to me if you have any questions on them, but carefully write up and turn in **only** *problems 1, 2, 3, 6, 7, 8, 9, and 11.* **Due: March 6**

1. Show the Euler characteristic of a tree is equal to 1.
   Hint: Induct on the number of vertices.

2. Show the Euler characteristic of a connected graph Γ is less than or equal to 1 with equality if and only if Γ is a tree.

3. Let Σ be a closed oriented surface and γ an embedded S^1 in Σ. If Σ' is obtained from Σ by surgery along γ, then prove that Σ' is orientable.

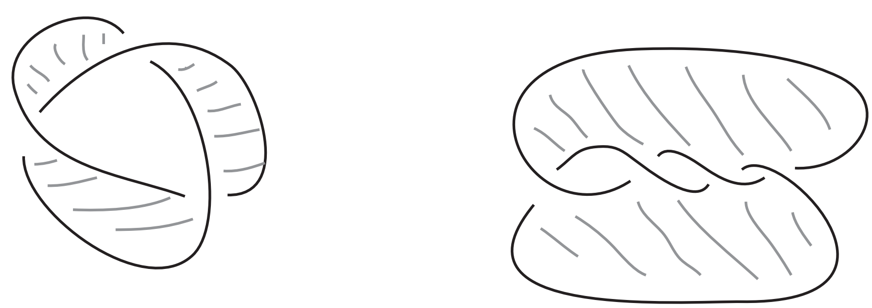
4. Given the surfaces in the previous problem and assume the curve γ does not separate Σ, show that Σ is homeomorphic to $\Sigma' \# T^2$. You may use the fact that a properly embedded arc in a surface with boundary has a neighborhood homeomorphic to $[-1,1] \times [0,1]$ so that the intersection of this neighborhood with the boundary is $[-1,1] \times \{0,1\}$.
   Hint: Recall you get Σ' from Σ by removing an annulus and gluing in two disks. When you remove the annulus from class we know you get a surface with two boundary components Σ'' that is connected (and path connected). So you can find an arc connecting the boundary components. Consider a neighborhood of the boundary and this arc before and after the surgery.

   Recall, $\Sigma_{n,m}$ denotes the connected sum of n tori with m disjoint disks removed and $N_{n,m}$ denotes the connected sum of n projective planes with m disjoint disks removed.

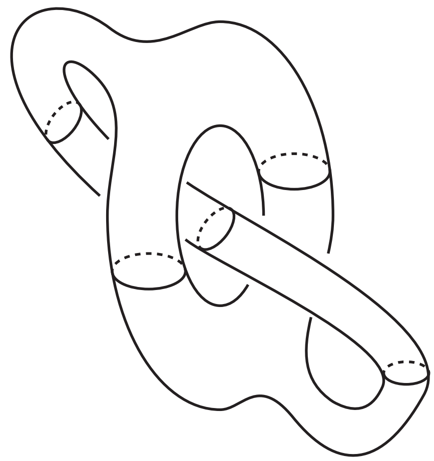
5. What surface in our classification is $N_3 \# \Sigma_2$ homeomorphic to?

6. What surface in our classification is $N_{1,1} \# \Sigma_{2,2}$ homeomorphic to?

7. Identify the surfaces the next figure and justify your answer.



8. Identify the surfaces the figure below and justify your answer.



9. Take a cubical block of wood (mathematically you can think of this as $[-1, 1] \times [-1, 1] \times [-1, 1]$). Take a drill and place it in the center of the top face, then drill a hold form the top to the bottom. Now take the drill and place it in the center of the right fact and then drill a hole from the right face to the left face. The boundary of this object is a surface. What surface is it?

10. If in the previous problem you had drilled the second hole so that it missed the first hole you drilled, what would the surface be?

11. Let X be a topological space. Show that the set of homeomorphisms of X form a group with multiplication given by composition.