Note Title 2/19/2016

## Compactness:

Reminder: $S$ is compact if $S \subset \bigcup_{i \in I} U_i$ with $U_i$ open then $\exists\ i_1, i_2, \dots, i_n$ such that

$$S \subset \bigcup_{j=1}^{n} U_{i_j}$$

Examples 2) $\mathbb{R}$ is not compact

$U_i = (-i, i)$ $\quad \mathbb{R} = \bigcup_{i=1}^{\infty} (-i, i)$ but $\mathbb{R} \not\subset \bigcup_{j=1}^{n} (-i_j, i_j)$

for any $i_1, \dots, i_n \in \mathbb{Z}$.

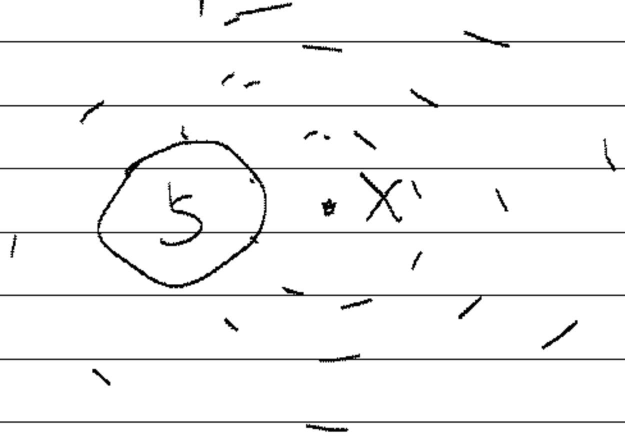
3) $(0,1)$ is not compact

$U_n = (\frac{1}{n}, 1)$ $\qquad$ $(0,1) = \bigcup_{n=1}^{\infty} (\frac{1}{n}, 1)$

$U_n = (-n, n)$ $\qquad$ $(0,1) \subset U_1$

Prop: $S$ compact $\Rightarrow$ $S$ bounded

proof:




Let $x \in E$

$$S \subset \bigcup_{r>0} B_r(x) = E$$

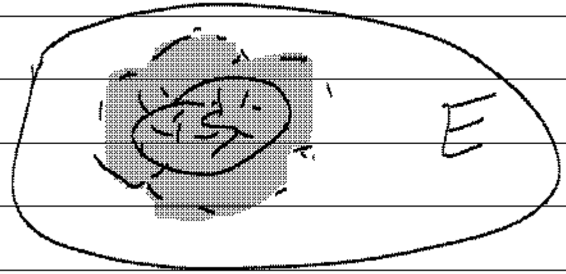
Since $S$ is compact, $\exists\ r_1, r_2, \dots, r_n$ such that $S \subset \bigcup_{i=1}^{n} B_{r_i}(x) =$

$= B_{\max(r_i)}(x)$ thus $S$ is

bounded

Prop: E compact. Let $S \subseteq E$, $S$ closed $\Rightarrow$ $S$ compact

proof: Let $U_i$ open for each $i \in I$.




Assume $S \subset \bigcup_{i \in I} U_i$. We need to show $\exists\, i_1, i_2, \ldots, i_n$ such that $S \subset \bigcup_{j=1}^{n} U_{i_j}$

$E \subset \bigcup_{i \in I} U_i \cup S^c$ note $S^c$ is open because $S$ is closed

Since $E$ is compact $\exists\, i_1, \ldots, i_n$ such that $E \subset \bigcup_{j=1}^{n} U_{i_j} \cup S^c$

$S \subset E \subset \bigcup_{j=1}^{n} U_{i_j} \cup S^c$

$S \subset \bigcup_{j=1}^{n} U_{i_j} \cup S^c$ $\qquad$ $S \cap S^c = \phi$ then

$S \subset \bigcup_{j=1}^{n} U_{i_j} \Rightarrow S$ is compact.