Suspension extraction through an opening before clogging

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Fluids with particles in suspension can be extracted through an orifice until particles bridge across the opening and clog it. We derive and study a mathematical model to predict the volume of fluid that can be extracted before clogging. Numerical simulations and closed-form upper and lower bound solutions show the dependency of the expected extracted volume on the volume fraction of particles in suspension and the relative particle-to-orifice size. These results are the first step toward a comprehensive understanding of the physical mechanisms leading to filter clogging with relevance to geomaterials, biomaterials, and industrial applications. © *2004 American Institute of Physics*. [DOI: 10.1063/1.1818342]

Fluid flow in porous media often involves the migration of fine particles. Under certain conditions, fine particles may clog the porous medium and cause a large decrease in hydraulic conductivity. Thus, it is important to understand the physical mechanisms that lead to clogging to try to design extraction procedures that prevent clogging. A relevant example is petroleum production where clogging leads to "formation damage" and marked loss of revenue. Clogging plays an important role in other mechanical and biological systems such as filters in water treatment, pharmaceutical processes, lungs and kidneys.

Various physical mechanisms converge to cause the clogging of a porous network.[1–11] The following simplified sequence of events underlies most clogging processes. The flow of the fluid phase drags fine particles; however, the rate of fines transport is lower than the rate of fluid transport because of electrical attraction to or collisions against pore walls. Retardation increases the local volume fraction of particles. If several migrating particles reach a pore throat simultaneously, the particles may bridge across the constriction and significantly lower the flow rate (pore throats are typically two to five times larger than the clogging migrating fines). In the vicinity of the clogged pore, the fluid velocity increases, promoting more collisions, retardation, and new bridging and clogging. Eventually, all fluid paths become clogged within an annular zone at some characteristic distance from the well.[11]

In this letter we study the following simple experiment that captures the main aspects of clogging at a single pore throat: A container is filled with a suspension made of an incompressible liquid and spherical particles. An opening is made in the container wall through which the suspension flows. The particles may or may not clog the opening. Our goal is to predict the volume of fluid extracted before clogging (if clogging does occur). The experiment described is illustrated in Fig. 1 (for clarity, the container and the particles are drawn in-plane as two-dimensional objects).

Our model relies on the following assumptions. The flow is not disturbed by the presence of particles. The center of each particle follows the fluid flow without retardation or advance. Before the opening is made, the center of each particle is randomly placed inside the container with a uniform probability distribution in space. The fluid and the particles are incompressible. Note that these assumptions allow particles to overlap.

We denote the area of the orifice by A and the volume fraction of particles by β. All the particles have the same radius r and volume $V_p = 4\pi r^3/3$.

For each point x in the container, we denote by $F(x)$ the volume of fluid extracted by the time the element of fluid initially at x reaches the opening. The left panel in Fig. 2 shows a two-dimensional sketch of level sets of the function F (the actual level sets of F are surfaces within the three-dimensional container). Due to the incompressibility of the fluid, the region enclosed by the level sets $\{x:F(x)=V+\Delta V\}$ and $\{x:F(x)=V\}$ has volume ΔV.

To motivate our criteria for clogging, assume that the fluid velocity is constant in space across the opening and out of the container. Once the volume of fluid initially in $\{x:V<F(x)\leq V+rA\}$ leaves the container, it forms a hypothetical cylinder with height r (see the right panel in Fig. 2). Since the centers of particles flow with the fluid, the number of centers of particles that belong to this cylinder is equal to the number of centers of particles initially placed in $\{x:V<F(x)\leq V+rA\}$. We denote this number by $k(V)$, i.e.,

$$k(V) = \text{number of particles initially placed in} \{x:V < F(x) \leq V + rA\}. \tag{1}$$

Note that the particles whose centers belong to this cylinder

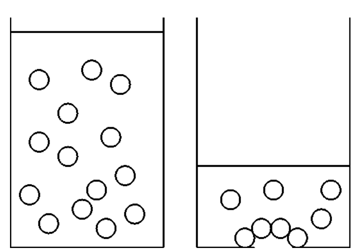

FIG. 1. Suspension in container. The left panel shows the container filled with the suspension before the opening is made. The right panel shows the system once clogging takes place.

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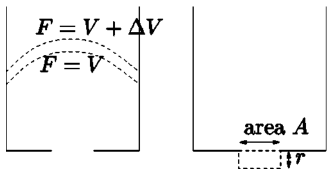

FIG. 2. The left panel is a two-dimensional sketch of level sets of F. The region enclosed by the dashed lines is $\{x: V < F(x) \leqslant V + \Delta V\}$. The right panel is a two-dimensional sketch of the three-dimensional cylinder (enclosed by dashed lines) which is formed by the suspension initially in $\{x: V < F(x) \leqslant V + rA\}$.

arrive "almost simultaneously" at the opening. We propose that clogging occurs when $k(V)$, the number of particles arriving almost simultaneously at the opening, exceeds a threshold $k_{\max}$. Thus, the volume of fluid extracted is

$$V^\star = \min_{\{V: k(V) > k_{\max}\}} V. \tag{2}$$

We define λ to be

$$\lambda = \frac{rA}{V_p} = \frac{3A}{4\pi r^2}. \tag{3}$$

Since the number of centers of particles that can belong to the cylinder under the condition that the particles do not overlap increases linearly with λ, we assume that $k_{\max}$ is of the form

$$k_{\max} = \gamma\lambda, \tag{4}$$

where γ is a parameter to be experimentally determined. Given a realization, Eqs. (1)–(4) determine the extracted volume $V^\star$. The numerical algorithm to compute $V^\star$ is described next.

Consider a large but finite number N of particles, forming a suspension of volume V_s, so that the volume fraction of particles is $\beta = NV_p/V_s$. The initial location of each particle is a random variable with uniform probability distribution. This fact along with the incompressibility of the fluid imply that the volume extracted by the time a particle reaches the opening is also a random variable with uniform probability distribution. As a consequence, if V_i is the volume extracted when the ith particle reaches the opening, these volumes V_i are the result of ordering N numbers selected independently with uniform probability distribution in the interval $[0, V_s]$. This is illustrated in Fig. 3.

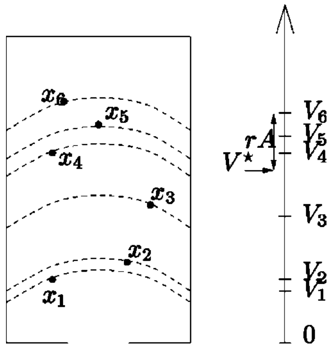

FIG. 3. The initial location of the ith particle to reach the opening is x_i. The dashed lines are the level sets of F. $V_i = F(x_i)$ is the volume extracted when the ith particle reaches the opening. $V^\star$ is the volume extracted before clogging.

Graphically, the algorithm developed to compute the extracted volume, given a suspension realization, consists of placing a segment of length rA on top of the vertical volume axis of Fig. 3 with the left end at 0, then moving the segment in the upward direction. As soon as the segment covers more than $k_{\max}$ particles simultaneously, clogging is predicted, and the location of the lower end of the segment is the extracted volume $V^\star$ prior to clogging. More precisely, for each i we define k_i to be the largest integer such that $V_{i-k_i+1} > V_i - rA$ (and $k_i \leqslant i$). If $i^\star = \min\{i: k_i > \gamma\lambda\}$ exists, clogging occurs and

$$V^\star = \begin{cases} 0 & \text{if } V_{i^\star} < rA \\ V_{i^\star} - rA & \text{if } V_{i^\star} \geqslant rA. \end{cases} \tag{5}$$

This observation leads to the following algorithm to compute $V^\star$ for a given realization

```
i ← 1
k ← 1
While k ≤ γλ and i < N
    i ← i+1
    k ← k+1
    While V_{i−k+1} ≤ V_i − rA
        k ← k−1
    end
end
If i = N and k ≤ γλ then
    V* ← V_s
else
    V* ← max{0, V_{i−k+1}}
end
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The expected extracted volume $E(V^\star)$ is computed by averaging the values obtained of $V^\star$ for a large number of different realizations.

In addition to the above-described numerical method, we have also obtained upper and lower bounds of the expected extracted volume. If the minimization of Eq. (2) is restricted to discrete values of $V = nrA$, where n is an integer (instead of all positive V), a closed form solution is obtained. The search for a minimum value in a subset renders a value equal to or larger than the true minimum. These arguments lead to the following upper bound of the expected extracted volume (the complete details are given in Ref. 12):

$$V_u = \frac{\mu_u}{1-\mu_u} rA,$$

where

$$\mu_u = e^{-\lambda\beta} \sum_{0 \leqslant k \leqslant \gamma\lambda} \frac{(\lambda\beta)^k}{k!}. \tag{6}$$

Given our criterion for clogging, the number of volumes V_i in the interval $(V^\star, V^\star + rA]$, where $V^\star$ is the volume extracted, is greater than $k_{\max}$ (see Fig. 3). Thus, if n is such that $nrA \leqslant V^\star < (n+1)rA$, then either the interval $(nrA, (n+1)rA]$ or the interval $((n+1)rA, (n+2)rA]$ contains at least $k_{\max}/2$ of the volumes V_i. Consequently, if m is the first integer for which $(mrA, (m+1)rA]$ contains $k_{\max}/2$ of the volumes V_i, then $V^\star \geqslant (m-1)rA$. This observation and similar techniques used to compute the upper bound lead to the following lower bound of the expected extracted volume (the complete analysis is given in Ref. 12)

TABLE I. Experimental and theoretical (with our numerical code and $\gamma = 1.05$) values of λ, for different values of particle volume fraction β, for which the extracted volume is 316 cm^3. The diameter of the particles is 0.3175 cm.

β	0.1	0.125	0.15	0.175	0.2
Experimental	3.9	4.3	4.7	5.1	5.5
Theoretical	3.2	4.0	5.0	5.4	5.9

$$V_\ell = \frac{\mu_\ell}{1-\mu_\ell} rA,$$

where

$$\mu_\ell = e^{-\lambda\beta} \sum_{0 \leqslant k \leqslant \gamma\lambda/2} \frac{(\lambda\beta)^k}{k!}. \tag{7}$$

A simple calculation shows that, in the parameter regime $\lambda\beta \ll 1$, the asymptotic behavior of the bounds is

$$V_u \simeq \frac{([\gamma\lambda]+1)!}{(\lambda\beta)^{[\gamma\lambda]+1}} rA, \quad V_\ell \simeq \frac{([\gamma\lambda/2]+1)!}{(\lambda\beta)^{[\gamma\lambda/2]+1}} rA, \tag{8}$$

where $[\gamma\lambda]$ is the integral part of $\gamma\lambda$, i.e., the largest integer that is not greater than $\gamma\lambda$. Equations (6) and (7) [and more explicitly, Eq. (8)] highlight the critical role of the relative particle-to-opening size (captured in λ) and the volume fraction β in determining the extracted volume.

Experimental results reported in Ref. 11 are simulated herein to infer the value of γ [see Eq. (4)]. The experimental procedure involves a 316 cm^3 cylinder filled with a suspension of spherical nylon spheres of radius $r = 1.58$ mm in water. Salt was added to the water until the fluid and particle mass densities matched and the suspension was made stable. The test started when a circular opening at the bottom of the cylinder was unplugged; the main observation was whether clogging developed during the 316 cm^3 flow through volume. The multiple tests conduced with various particle volume fractions β and opening diameters permitted defining a boundary between clogging and not-clogging conditions in terms of relative size [captured in λ defined in Eq. (3)] and volume fraction β. We simulated these results using the numerical code and obtained a least-squares fit for $\gamma = 1.05$. Measured and predicted λ values are compared in Table I for different volume fractions β. Note that the smaller the particle volume fraction β, the smaller the relative opening-to-particle size must be (i.e., λ) to observe clogging within a given flow through volume. We anticipate that γ is a material parameter that depends on particle geometry and the ensuing inter-particle friction and interlocking.

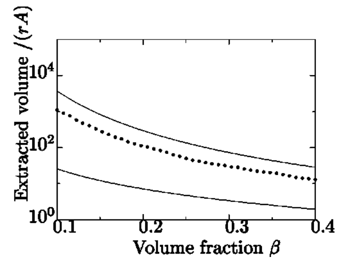

FIG. 4. Normalized expected extracted volume $E(V^\star)/(rA)$ (dotted line) normalized upper bound $V_u/(rA)$ (upper solid line) and normalized lower bound $V_\ell/(rA)$ (lower solid line) vs volume fraction β (for $\lambda=3$ and $\gamma=1$).

In Fig. 4, the expected extracted volume $E(V^\star)$ and the upper and lower bounds V_u and V_ℓ are plotted versus the volume fraction β, for a value of $\lambda=3$. The expected extracted volumes were numerically computed with the method described in this letter. Note that, for a circular opening, $\lambda = 3$ when the radius of the orifice is twice the radius of the particles.

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