

FINAL EXAM

Time: 120hrs

1. Let n be a prime. Is $(\mathbf{Z}_n - \{[0]\}, \odot)$ a group? Why?
2. What is the smallest non-abelian group? Why?
3. Is the collection of all infinite sets of natural numbers a countable set? Why?
4. A graph has q edges, where $q \geq 2$. What is the smallest number of vertices that the graph can have? Why?
5. Let $g: A \rightarrow B$, and $f: B \rightarrow C$ be functions. prove that if $f \circ g: A \rightarrow C$ is one-to-one, then $g: A \rightarrow B$ is also one-to-one. Is the converse true? Why?
6. What is the solution to the system of congruences: $x \equiv 1 \pmod{3}$, $x \equiv 2 \pmod{5}$, and $x \equiv 3 \pmod{7}$? Why?
7. Is the sum of any three consecutive positive cubes a multiple of 9? Why?
8. Is a rectangle with unequal sides less symmetric than a square? Why?
9. How many pairs of prime numbers p and q are there such that $q - p = 3$? Why?
10. Is the set of prime numbers finite? Why?

Each problem is worth 10 pts.