Midterm 2
Time: 50 minutes

Show all work and justify your answers.

1. Compute the rank of the following matrix and decide whether it is invertible. If so, find its inverse: $A = \begin{bmatrix} 1 & 1 & -1 \\ 1 & -1 & 2 \\ 1 & 2 & 0 \end{bmatrix}$.

2. Find the least square solution to $A\mathbf{x} = b$, where $A = \begin{bmatrix} 1 & 1 & 2 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix}$ and $b = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$.

3. Consider the system of equations

$$
\begin{aligned}
x - 2y + az &= 2 \\
x + y + z &= 0 \\
3y + z &= 2.
\end{aligned}
$$

For which values of a, if any, does this system have **(a)** a unique solution? **(b)** no solution? **(c)** infinitely many solutions?

4. Find the interval of convergence of

$$\sum \frac{1}{k} x^k.$$

Each problem is worth 25pts.