

PRACTICE QUIZ 1

1. Recall that a function f is said to be *odd* if $f(-x) = -f(x)$ and *even* if $f(-x) = f(x)$. Classify the following functions as even, odd, both, or neither: x , x^2 , x^3 , $\sin x$, $\cos x$, and e^x .
2. Show that any function which is both even and odd must be identically zero, i.e., $f(x) = 0$ for all x .
3. Show that any function which is neither even nor odd may be written as the sum of an odd and an even function neither of which is identically zero (*Hints*: follow these steps
 - (i) Define $E(x) := \frac{f(x)+f(-x)}{2}$ and $O(x) := \frac{f(x)-f(-x)}{2}$. Show that if f is neither even nor odd, then the functions $E(x)$ and $O(x)$ are not identically zero.
 - (ii) Show that $E(x) + O(x) = f(x)$.
 - (iii) Check that $E(x)$ is even and $O(x)$ is odd.
4. Write e^x as the sum of an odd and an even function and graph all three functions. (*Note*: The odd and even parts of e^x are important functions which are known, respectively, as the hyperbolic sine (\sinh) and hyperbolic cosine (\cosh).