Math 19B Jun 6, 2000
Calculus for Science, Engineering, and Mathematics
Spring 2000, UCSC

FINAL EXAM

Time: 180min

1. Evaluate:

a) $\int \tan\theta \, d\theta$ **b)** $\int \ln 2x \, dx$ **c)** $\int \frac{1}{1-x^2} \, dx$ **d)** $\int e^{\sqrt{x}} \, dx.$

2. Find the area of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$.

3. Compute the surface area of a zone of a sphere of radius R which is contained between a pair of parallel planes separated by a distance of d, assuming that both planes intersect the sphere.

4. Compute the center of mass of a half-disk of radius R, and use this, via Pappus's theorem, to find the volume of a sphere.

5. Evaluate $\int_1^\infty \frac{1}{x^p} \, dx$, and use it to decide for which values of p the series $\sum_{n=1}^\infty \frac{1}{n^p}$ converges, and for which values it diverges.

6. Test the series for convergence or divergence:

a) $\sum_{n=1}^{\infty} \frac{n}{n+1}$ **b)** $\sum_{n=2}^{\infty} \frac{(-1)^n}{\ln n}$ **c)** $\sum_{n=1}^{\infty} \frac{2^n}{n!}$ **d)** $\sum_{n=1}^{\infty} \frac{3^n}{5^n + n}.$

7. **a)** Find the Taylor series for e^x and compute its radius of convergence. **b)** Estimate the numerical value of e accurate to within 1/100.

8. **a)** Find a power series for $\tan^{-1} x$, and use it to obtain a series for π. **b)** How many terms in this series must be added in order to estimate the numerical value of π accurate to within 1/100.

Problems 1 *and* 6 *are worth* 20 *points each (*5 *points for each part), and the rest of the problems are worth* 10 *points each.*

LaTeX ..... MG