

Midterm 2

Time: 60 minutes

1. Find the function with gradient $\mathbf{F}(x, y) = 2xy\mathbf{i} + (1 + x^2)\mathbf{j}$.

2. A cylindrical can has prescribed surface area S . What dimensions for the can yield the maximum volume?

- 3.** Find the volume of the “ice cream cone” region bounded inside the sphere $x^2 + y^2 + z^2 = 1$ and above the cone $z = \sqrt{x^2 + y^2}$.

4. Find the area of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$.

5. Find center of mass of half a ball of radius 1, i.e., the region bounded inside the sphere $x^2 + y^2 + z^2 = 1$ and above the plane $z = 0$.