MIDTERM   **Time: 120min**

Choose 5 of the following 8 problems. Wrrite complete answers.

1. Show that if a closed planar curve lies inside a circle of radius r then its curvature is bigger than or equal to $1/r$ at some point.

2. Show that if the curvature of a planar curve is monotone, then it has no self intersections.

3. Compute the curvature and torison of the helix $(r\cos(t), r\sin(t), ht)$, where r and h are constants.

4. Show that if the principal normals of a planar curve all pass through the same point, then the curve is a circle.

5. Show that the tantrix of a closed curve intersects every great circle.

6. Let $\alpha\colon I \to \mathbf{R}^3$ be a unit speed curve whose torsion never vanishes. Suppose that the binormal vector $B\colon I \to \mathbf{S}^2$ is known. Show that we can then recover the curvature and torsion of α.

7. Suppose that $\alpha\colon I \to \mathbf{R}^2$ is a closed curve such that for any constant s, $\|\alpha(t+s) - \alpha(t)\|$ is constant for all $t \in I$. Show that α is a circle.

8. Show that the only closed planar curve of constant curvature is a circle.

Each problem is worth 20pts.