

Midterm 1

Time: 50min

1. Find a vector of length 5 that points in the direction opposite to $2\mathbf{i} + \mathbf{j} - \mathbf{k}$.
2. Show that the diagonals of a parallelogram have the same length if and only if the parallelogram is a rectangle.
3. Find the area of the triangle with vertices $(1, 0, 1)$, $(3, 0, 1)$, $(1, 3, 1)$.
4. If $a \times b = \mathbf{i} + \mathbf{j} + \mathbf{k}$, what is $(2a + b) \times (a - 3b)$?
5. Find the distance between the point $(2, -1)$ and the line $\ell: x = 3t + 7, y = 5t - 3$.
6. What is the distance between the two planes
 $5x - 2y + 2z = 12$ and $-10x + 4y - 4z = 8$.
7. Find a formula for the distance between the planes $Ax + By + Cz = D_1$ and $Ax + By + Cz = D_2$.
8. Show that if a path $x(t)$ lies on a sphere, then $x(t)$ is always perpendicular to its derivative.
9. Show that for any three real numbers a, b, c , we have:

$$\frac{a + b + c}{3} \leq \sqrt{a^2 + b^2 + c^2}.$$

Each problem is worth 12pts.