HOME WORK 3 ANALYSIS 2

Due March 4. All the problems marked with an asterisk are optional.

1. Show that in any metric genrated by a norm in $\mathbb{R}^n$, if set is closed and bounded, then it is compact.

2. Show a continuous function on a compact attains its maximal and minimal value.

3. Find an example of a continuous, but not uniformly continuous function on the open unit ball B_2^n.

4. Find an example of a sequence f_n of continuous functions converging pointwise to a discontinuous function.

5. Prove that uniform limits preserve continuity.

6^***.** Let $f : [0, 1] \to \mathbb{R}$ be a continuous integrable function and suppose that

$$\int_0^1 x^n f(x) dx = 0$$

for any $n \in \mathbb{N}$. Show that f must equal to zero everywhere on $[0, 1]$.

7. Verify that the uniform limit of

$$f_n(x) = \sum_{n=1}^{\infty} 4^{-n} \cos(32^n \pi x)$$

on $[0, 1]$ is a continuous function which is not differentiable at any point.

8. Show that the convolution of continuous functions is a continuous function.