HOME WORK IV, DIFFERENTIAL GEOMETRY: RIEMANNIAN METRIC, AFFINE CONNECTIONS AND GEODESICS.

Due February 28. The Home Work must be uploaded on Canvas as a pdf. To complete the home work, use the lecture notes, as well as the book of Chavel "Riemannian geometry: a modern introduction", Chapters 1.1, 1.2, 1.5, 1.6, 1.7 and the lecture notes 12, 13 and 14 of Mohammad Ghomi https://people.math.gatech.edu/~ghomi/LectureNotes/index.html. **Questions marked with an asterisk are optional. Please contact me if you have any questions!**

1. Let $d(p, q)$ be the distance generated on a Riemannian manifold M by the Riemannian metric g. Show that d is positive definite, thereby completing the proof that a Riemannian manifold is a metric space. That is, show that if $d(x, y) = 0$ then $x = y$.

2.* Suppose M is an $n-$dimensional differentiable manifold with topology induced from $\mathbb{R}^n$. Show that this topology coincides with the one generated by a Riemannian metric generated by g on M. (Recall that a topology is said to be generated by a metric $d(x, y)$ if it is generated by the open metric balls $B(x, R) = \{y \in M : d(x, y) < R\}$.)

3. Consider M to be the graph of the function $z = xy$ in $\mathbb{R}^3$, with the induced (from $\mathbb{R}^3$) connection ∇. Compute the Christofel symbols at $(0, 0, 0)$.

4. Give an example (describe it locally) of a geodesic curve passing through $(0, 0, 0)$ on the manifold M given by the graph of a function $z = xy$ in $\mathbb{R}^3$, with the connection induced from $\mathbb{R}^3$.