ANALYSIS I, HOME WORK 5, FALL 2018

Due October 24.

1. Show that any decreasing sequence a_n in $\mathbb{R}$, which is bounded from below, is convergent.

2. Prove that $\lim_{n\to\infty} \min(a_n, b_n) = \min(\lim_{n\to\infty} a_n, \lim_{n\to\infty} b_n)$.

3. Show that there exists a positive real number x such that $x^4 = 5$.

4.* Does there exist a sequence with infinitely many limit points? Give an example if the answer is yes.

5. Let a_n and b_n be two sequences of real numbers such that $a_n \leq b_n$ for all natural n. Show that $\limsup_{n\to\infty} a_n \leq \limsup_{n\to\infty} b_n$.

6. For $x \in \mathbb{R}$, does $\lim_{n\to\infty} x^n$ exist, and if so, what does it equal to? In class we studied that case when $x \in (0, 1)$; consider, with the proof, all the necessary cases.