HOME WORK 5, ANALYSIS II

Due April 15. Problems marked with asterisk are optional, but highly recommended. Please contact me if you have any questions!

1. Please write the full proof of the multivariable chain rule (Theorem 6.4.1).

2*. Find an example of a function which satisfies all the assumptions of Clairaut's theorem with the exception of continuity of second derivatives, and such that the conclusion of Clairaut's theorem fails.

3*. Please derive the Implicit function theorem from the Inverse mapping theorem.

4. Let $A \subset \mathbb{R}^n$ and $B \subset \mathbb{R}^k$. Show that $m^*_{n+k}(A \times B) \leq m^*_n(A) m^*_k(B)$. Here m^*_p stands for the outer measure in $\mathbb{R}^p$.

5. Check that any half-space in $\mathbb{R}^n$ is Lebesgue-measurable. (recall that a half-space is a set of the form $\{x \in \mathbb{R}^n : \langle x, v \rangle \leq \rho\}$, for some $\rho \in \mathbb{R}$ and $v \in \mathbb{R}^n$.)