HOME WORK 8, ANALYSIS I

Due December 3. Problems marked with asterisk are optional, but highly recommended. Please contact me if you have any questions!

1^*. Give an example of a function on $\mathbb{R}$, continuous at all irrational points but discontinuous at the rational points. Provide all the proofs.

2^*. Give a second proof of the fact that any continuous function on $[0, 1]$ is uniformly continuous: use the exercise 4 from Home work 7 about finite subcover property of compacts.

3. Prove that f is uniformly continuous on A if and only if, for any pair of equivalent sequences $\{x_n\} \subset A$ and $\{y_n\} \subset A$, the sequences $\{f(x_n)\}$ and $\{f(y_n)\}$ are necessarily equivalent.

4. Find derivatives of

a) $x^2 + 3x - 1$ at $x_0 = 5$;

b) $\cos \frac{x^2}{x+1} + \sin \frac{1}{x^2}$ at $x_0 = 1$;

c) $\sqrt{x^3 - \sin^2(3x)}$ at $x_0 = 10$.

5. Can it happen, that a function is differentiable at x_0, and has derivative equal to zero at x_0, but x_0 is neither local maxima nor local minima?

6. Show that if $f : \mathbb{R} \to \mathbb{R}$ is everywhere differentiable, and the function f' is bounded, then f is uniformly continuous.

7. Give an example of a function on $[-1, 1]$ which is continuous, monotone, but not differentiable at $x = 0$.