

HW PROBLEMS SET 3: ARGUMENT BY CONTRADICTION, INDUCTION, PIGEONHOLE PRINCIPLE

1. Given nine points inside the unit square, prove that some three of them form a triangle whose area does not exceed $\frac{1}{8}$.
2. Each of nine straight lines divides a square into two quadrilaterals with the ratio of their areas equal to $r > 0$. Prove that at least three of these lines are concurrent.
3. Show that any convex polyhedron has two faces with the same number of edges.
4. The points of the plane are colored by finitely many colors. Prove that one can find a rectangle with vertices of the same color.
5. Let n be a fixed positive integer. How many ways are there to write $n = a_1 + a_2 + \dots + a_k$, with k an arbitrary positive integer and $a_1 \leq a_2 \leq \dots \leq a_k$? For example, with $n = 4$ there are four ways: 4, 2+2, 1+1+2, 1+1+1+1.
6. Shanille O'Keal shoots free throws on a basketball court. She hits the first and misses the second, and thereafter the probability that she hits the next shot is equal to the proportion of shots she has hit so far. What is the probability she hits exactly 50 of her first 100 shots?
7. Let S be a convex set in the plane that contains three noncollinear points. Each point of S is colored by one of p colors, $p > 1$. Prove that for any $n \geq 3$ there exist infinitely many congruent n -gons whose vertices are all of the same color.