MATH 4317 (C. Heil) HOMEWORK #2 DUE: June 7, 2007

Work the following problems and hand in your solutions. You may work together with other people in the class, but you must each write up your solutions independently. A subset of these will be selected for grading. Write LEGIBLY on the FRONT side of the page only, and STAPLE your pages together.

1. Problem 7 G. If $I_n = [a_n, b_n]$, $n \in \mathbf{N}$, is a nested sequence of closed cells, show that for every $m$ and $n \in \mathbf{N}$ we have

$$a_1 \le a_2 \le \cdots \le a_n \le \cdots \le b_m \le \cdots \le b_2 \le b_1.$$

If we put $\xi = \sup\{a_n : n \in \mathbf{N}\}$ and $\eta = \inf\{b_m : m \in \mathbf{N}\}$, show that $[\xi, \eta] = \bigcap_{n \in \mathbf{N}} I_n$.

2. Let $F$ be the Cantor set constructed in class. Prove that there is no interval $(a, b)$ that is contained in $F$. Specifically, show that if $0 < a < b < 1$, then $(a, b)$ is not a subset of $F$.

3. Problem 8 L (refer to 8 F and 8 G for definitions). Show that there exist positive constants $a$, $b$ such that

$$a\,\|x\|_1 \le \|x\|_\infty \le b\,\|x\|_1 \qquad \text{for all } x \in \mathbf{R}^p.$$

Find the largest constant $a$ and the smallest constant $b$ with this property.

4. Problem 8 Q. A subset $K$ of $\mathbf{R}^p$ is said to be **convex** if, whenever $x$, $y$ belong to $K$ and $t$ is a real number such that $0 \le t \le 1$, then the point

$$(1-t)x + ty = x + t(y - x)$$

also belongs to $K$. Interpret this condition geometrically and show that the subsets

$$K_1 = \{x \in \mathbf{R}^2 : \|x\| \le 1\},$$

$$K_2 = \{(\xi, \eta) \in \mathbf{R}^2 : 0 < \xi < \eta\},$$

$$K_3 = \{(\xi, \eta) \in \mathbf{R}^2 : 0 \le \eta \le \xi \le 1\},$$

are convex but that the subset

$$K_4 = \{x \in \mathbf{R}^2 : \|x\| = 1\}$$

is not convex.