MATH 4317 (C. Heil) HOMEWORK #3 DUE: June 26, 2007

Work the following problems and hand in your solutions. You may work together with other people in the class, but you must each write up your solutions independently. A subset of these will be selected for grading. Write LEGIBLY on the FRONT side of the page only, and STAPLE your pages together.

1. Problem 9 L. If A is any subset of $\mathbf{R}^p$, let A^- denote the intersection of all closed sets containing A; the set A^- is called the **closure** of A. Note that A^- is a closed set; prove that it is the smallest closed set containing A. Prove that

$$A \subseteq A^-, \quad (A^-)^- = A^-, \quad (A \cup B)^- = A^- \cup B^-, \quad \emptyset^- = \emptyset.$$

Give an example to show that $(A \cap B)^- = A^- \cap B^-$ may not hold.

NOTE: Part of this problem asks you to prove that A^- is the smallest closed set containing A. To do this, you must show that A^- is closed, and that if B is any closed set such that $A \subseteq B$, then $A^- \subseteq B$.

2. Problem 10 G. Show that every point in the Cantor set F is a cluster point of both F and $\mathcal{C}(F)$.

3. Problem 11 D. Prove that if K is a compact subset of $\mathbf{R}$, then K is compact when regarded as a subset of $\mathbf{R}^2$.

NOTES: The statement "when K is regarded as a subset of $\mathbf{R}^2$" means that you are to prove that the set

$$K' = \{(x, 0) : x \in K\}$$

is a compact subset of $\mathbf{R}^2$. Do NOT use the Heine–Borel Theorem in your proof; instead, prove that K' is compact by directly using the definition of compact set.

4. Problem 12 B. If $C \subseteq \mathbf{R}^p$ is connected and x is a cluster point of C, then $C \cup \{x\}$ is connected.