MATH 4317 (C. Heil) HOMEWORK #5 DUE: July 26, 2007

Work the following problems and hand in your solutions. You may work together with other people in the class, but you must each write up your solutions independently. A subset of these will be selected for grading. Write LEGIBLY on the FRONT side of the page only, and STAPLE your pages together.

1. Problem 18 F. Let $(x_n)$ be a bounded sequence of strictly positive real numbers. Show that $\limsup (x_n^{1/n}) \le \limsup (x_{n+1}/x_n)$.

Hint: Let $s = \limsup (x_{n+1}/x_n)$. Choose $\varepsilon > 0$. Then there exist only finitely many $n$ such that $x_{n+1}/x_n > s + \varepsilon$. Hence there is an $N$ such that $x_{n+1}/x_n \le s + \varepsilon$ for all $n \ge N$.

2. Problem 18 I. Show that $\limsup (x_n) = \infty$ if and only if there is a subsequence $(x_{n_k})$ such that $\lim_{k\to\infty} x_{n_k} = \infty$.

3. Problem 20 A. Prove that if $f$ is defined for $x \ge 0$ by $f(x) = \sqrt{x}$, then $f$ is continuous at every point of its domain.

4. Problem 20 E. Let $f$ be the function on $\mathbf{R}$ to $\mathbf{R}$ defined by

$$f(x) \;=\; \begin{cases} x, & \text{if } x \text{ is irrational},\\ 1-x, & \text{if } x \text{ is rational}. \end{cases}$$

Prove that $f$ is continuous at $x = 1/2$ and discontinuous elsewhere.