

Work the following problems and hand in your solutions. You may work together with other people in the class, but you must each write up your solutions independently. A subset of these will be selected for grading. Write LEGIBLY on the FRONT side of the page only, and STAPLE your pages together.

1. Show that if  $f: \mathbb{R}^d \rightarrow \mathbb{R}$  is continuous, then its essential supremum coincides with its supremum in the usual sense, i.e.,

$$\operatorname{ess\,sup}_{x \in \mathbb{R}^d} f(x) = \sup_{x \in \mathbb{R}^d} f(x).$$

2. Lecture notes PROBLEM 1.20 (on p. 23).

3. Lecture notes PROBLEM 1.32 (on p. 33).

4. Assume  $g: \mathbb{R}^n \rightarrow \mathbb{R}^m$  is continuous. Show that if  $K \subseteq \mathbb{R}^n$  is compact, then  $g(K) \subseteq \mathbb{R}^m$  is compact. Give a counterexample that shows that we cannot replace the word “compact” by “closed” in this problem.